BCCA

SCM & DM

PAPER 16

CMA FINAL

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INDEX

Train yourself as if someone out there is getting trained harder than you to kill you.

- **BRUCE LEE**

DECISION MAKING (WEIGHTAGE 40% , SULLABUS 2022)			
INDEX	WEIGHT	PAGE NO	
6. Linear programme planning	15%		
7. Transportation			
8. Assignment			
9. Game theory	15%		
10. Simulation			
11. Network analysis			
12. Learning curve			
13.maxima & minima	10%		
14. Business forecasting			
15. Data analytics			

COLOURS

BLUE – CONTENT

RED – IMPORTANT POINT / KEY WORD

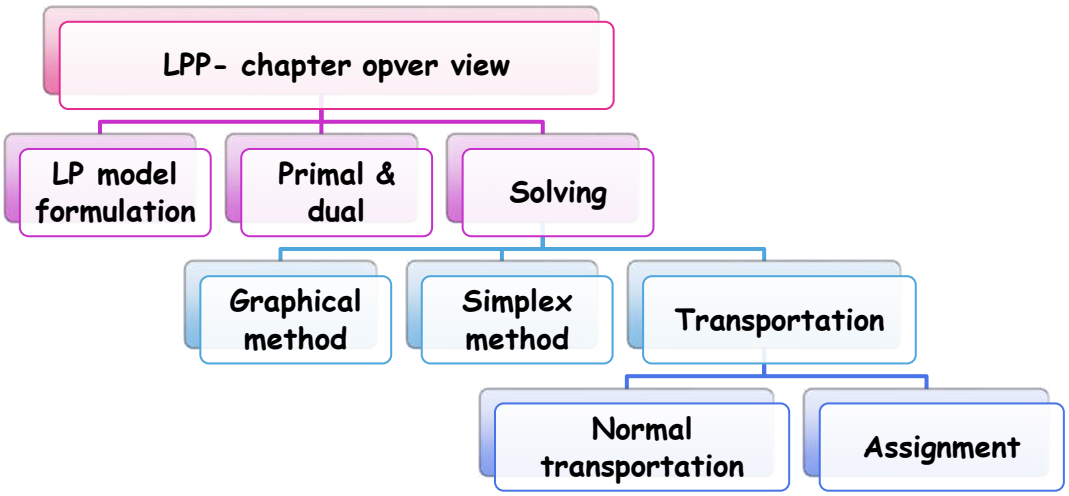
BLACK – HEADINGS - bold

GREEN – FORMULAE

SHORT CUTS;

SMQ -ICMAI STUDY MATERIAL QUESTION.

PAQ – PREVIOUS ATTEMPT QUESTIONS



Introduction

1) Example:

Products	X	Y
Contribution Rs.	4	6
Raw Material Kgs per unit	1 Kg.	2 Kg.
Contribution /Kg.	4	3
Rank based	I	II
Labour Hours per unit	2 Hours	2 hours
Contribution per hour	2	3
Rank	II	I

2) When something limits our production it is called "**Limiting Factor**" which may be raw material resource availability or labour hour capacity etc.,

3) When there exists a limiting factor it should be allocated to the most profitable product and the profitability is ranked using the contribution per limiting factor.

4) In the above example, there are two limiting factors (i) Raw Material - 100 Kgs (ii) Labour Hours -100 Hours

5) Raw Material as a limiting factor selects Product X and labour hour Product Y. Thus there exists aconflict in ranking. In such a case the problem should be solved using LPP technique (Linear Programming Problem) technique.

6) This chapter will be applied when we have an objective to be achieved with multiple constraints.

LP model formulation

7) The above situation can be formulated into a Linear Programming Problem as follows:Let X_1 be number of units of Product 'X' and X_2 be number of units of

Objective function

Product 'Y'. Max $Z = 4X_1 + 6X_2$

Constraints

Subject to $X_1 + 2X_2 < 100$

$2X_1 + 2X_2 < 100$

Non-negative constraints

Where X_1 & X_2 > 0 (always non negative constraints)

Objective Function

8) The first equation (Max Z = $4X_1 + 6X_2$) is called **"Objective Function"** which can be of twotypes:
   a) Maximization – Maximize production, profit, revenue, etc.,
   b) Minimization – Minimize cost, time etc.,

Constraints

9) The objective function should be achieved subject to constraints which can be of 3 types:
   a) Not more than (<)
   b) At least (>)
   c) Exactly (=)
10) The items for which we should calculate values are called **"Variables"** which are represented asvariables (Z, X_1, X_2 etc.,)
11) The numbers prefixed to the variables are called **"Coefficients"**. For example, coefficient of X_2 in objective function is '6'.
12) We have to find out the value of variables to achieve our object and this can be done by solving theLPP (Linear Programming Problems) under two approaches
   a) Graphical Approach
   b) Simplex Method

Graphical approach can be used only for two variable problems and that to it can give only solution and not wealth of information which simplex can do.

Format to solve DUAL problem

		Primal (minimization)			
Dual (maximization)	Decision variables	X_1	X_2	relation	RHS of constraints
	Relation				
	RHS of constraints				

	Primal	**Dual**
Objective function	Maximization	Minimization
	Minimization	Maximization
Constraints	No of constraints	No of variables
variables	No of variables	No of constraints
Relation	≥	≤
	≤	≥
	RHS of constraints	Co efficients of variables in objective function
	Co efficients of variables in objective function	RHS of constraints

Graphical method

Simplex method

Illustration-1

A shopkeeper deals in two items – Wall hangings and Artificial plants. He has ` 50000 to invest and a space to store 100 pieces at the most. Costs of Wall hangings and Artificial plants are respectively ` 450 and ` 200 each. Hecan sell a Wall hanging at a profit of ` 80 and an Artificial plant at a profit of ` 37. Assuming that he can sell all the items that he buys, formulate a Linear Programming problem in order to maximize his profit.

Illustration-2

A dealer of cement has two warehouses M and N with stocks of 30000 and 20000 bags of cement respectively. Three customers A, B and C have placed order on the dealer for 15000, 20000 and 15000 bags respectively. Costs of transportation per 1000 bags of cement from different warehouses to different customers are given below.

	Transportation Cost (00) per 1000 bags		
To	A	B	C
From			
M	40	20	20
N	20	60	40

The dealer wants to find how to fulfill the orders so that the transportation cost is minimum. Formulate the problem.

Illustration-3

One kind of cake requires 200 grams of flour and 25 grams of fat and another kind of cake requires 100 grams of flour and 50 grams of fat. Find the maximum number of cakes that can be made from 5 kgs. of flour and 1 kg. of fat assuming there is no shortage of other ingredients required for making cakes. Formulate LPP based on the information given and solve graphically.

Illustration-4

Mr. Lal is on a low cholesterol diet. During lunch at the office canteen he always chooses between two particular types of meal – Type A and Type B. The table below lists the amount of protein, carbohydrates and vitamins each meal provides along with the amount of cholesterol (which he is trying to minimize). He needs at least 200 grams of protein, 960 grams of carbohydrates and 40 grams of vitamins for lunch each month. Over this time period, how many days should he have Type A meal and how many days the Type B meal so that he gets adequate amount of protein, carbohydrates and vitamins and at the same time minimizes his cholesterol intake? Use Graphical Method.

	Type A meal	Type B meal
Protein (Grams)	8	16
Carbohydrates (Grams)	60	40
Vitamins (Grams)	2	2
Cholesterol (Miligrams)	60	50

Illustration-5

Solve graphically the following LPP –

Maximize $M = 50x_1 + 60x_2$

subject to the constraints $2x_1 + x_2 \leq 300$, $3x_1 + 4x_2 \leq 509$, $4x_1 + 7x_2 \leq 812$, $x_1 \geq 0$, $x_2 \geq 0$

Illustration-6

A firm manufactures and sells two products Alpha and Beta. Each unit of Alpha requires 1 hour of machining and 2 hours of skilled labour, whereas each unit of Beta uses 2 hours of machining and 1 hour of labour. For the coming month the machine capacity is limited to 720 machine hours and the skilled labour is limited to 780 hours. Not more than 320 units of Alpha can be sold in the market during a month.

(i) Develop a suitable model that will enable determination of the optimal product mix.

(ii) Determine the optimal product-mix and the maximum contribution if Unit contribution from Alpha is ` 6 and from Beta is ` 4.

(iii) What will be the incremental contribution per unit of the machine hour, per unit of labour, per unit of Alpha saleable?

Illustration-7

Sri Lanka, the third largest tea producing country has a production share of 9% of the international market and one of the world`s leading exporters with a share of 19% of the global demand. Thus tea industry is crucial to enhance their economic competitiveness in the world market. The nature of the highly competitive global market has made scientific and reasonable production management increasingly important for tea companies to differentiate themselves from competitors. In order to enhance their competitive position, Sri Lankan tea manufacturers are giving serious thought to use optimization techniques like Linear Programming to find their best product mix to achieve maximization of profit. Dulwan Tea Company, established in 1974 is one of the leading tea exporters of the country. They use their own leaves which grow in their tea plantations. More than 2500 varieties of flavored and non-flavored tea products are produced and globally exported by the

company. This brand is available in more than 90 countries in the world including UK, Poland, Canada, South Africa, Australia and New Zealand. Therefore how to optimize the production process yielding maximum profit is a critical and challenging task in front of the decision makers of Dulwan. After lot of deliberations among themselves, the management of Dulwan has decided to hire a Cost and Management consultant.

Accordingly they hired Mr. Kuppuswamy, a resident of Jafna, Sri Lanka and a well-known consultant of the island. In his first visit to the company the management explained to him the requirements and Mr. Kuppuswamy technically phrased the objective of the work as follows.

- To formulate a mathematical model that would suggest a viable product mix to ensure maximum profit of the company as well as evaluating performance of the proposed product mix.
- To highlight the peculiarities of using linear programming technique at a single operating procedure and prove that despite the obstacles, the application of the technique in determining the product mix enables Dulwan Tea Company to be more profitable than the otherwise.

Thereafter a team is formed from the existing employees of the company and under the guidance of Mr. Kuppuswamy they started working to formulate the problem as a Linear Programming model. Since the company is dealing with huge varieties of tea product, everybody could realize that solving such LPP manually is impossible. So it is decided to purchase a suitable software for the purpose and Mr. Kuppuswamy is requested to get at least three quotes from renowned global software companies. When the process is on, all of a sudden new opportunities open and the company decided to bid for supplying few of its very premium quality tea to the European market. But the management was not very sure as to which quality of tea they should try to sell so that the objective of profit maximization is fulfilled. Once again Mr. Kuppuswamy was approached and this time he decided to find the best product mix by solving the problem manually (as variety of very premium quality tea was not much and also the decision regarding which software to purchase not finalized).

During solution of the problem manually, at one stage the following Simplex Table is obtained

C_B	Product Mix	Quantity	x_1	x_2	x_3	s_1	s_2	s_3	A_1
2	x_1	4	1	2	1/2	0	0	1/4	0
0	s_2	12	0	0	-1	0	1	-1/2	0
0	s_1	12	0	6	0	1	0	1	-1
	C_j		2	4	1	0	0	0	- M
	Z_j	8	2	4	1	0	0	$\frac{1}{2}$	0
	$C_j - Z_j$		0	0	0	0	0	-1/2	-M

Answer the following questions, with proper explanation, related to the Simplex Table above.

i) How many varieties of very Premium quality tea are considered in the problem?
ii) Is the solution given in the Table above Optimal?
iii) What is the Objective Function?
iv) Is there any alternate solution to the problem?
v) Is the solution feasible?
vi) What is the optimum product mix and the maximum profit.
vii) If any alternate solution is possible then find it.

Illustration-8

A company possesses two manufacturing plants each of which can produce three products X, Y and Z from a common raw material. However, the proportions in which the products are produced are different in each plant andso are the plant's operating costs per hour. Data on production per hour costs are given below, together with currentorders in hand for each product.

	Product			Operating cost perhour in `
	X	Y	Z	
Plant A	2	4	3	9
Plant B	4	3	2	10
Orders on hand	50	24	60	

You are required to use the simplex method to find the number of production hours needed to fulfill the orders on hand at minimum cost.
Interpret the main features of the final solution

Illustration-9

Obtain the Dual of the following Primal Problem.
Minimize $Z = 6000x_1 + 4000x_2$ subject to the constraints $4x_1 + x_2 \geq 12$, $9x_1 + x_2 \geq 20$, $7x_1 + 3x_2 \geq 18$,
$10x_1 + 40x_2 \geq 40$ and x_1 & $x_2 \geq 0$

Illustration-10

Write the Dual of the following –
Maximize $Z = 5x_1 + 10x_2$ subject to $2x_1 - 3x_2 \leq 7$, $x_1 + 2x_2 = 4$ and x_1 , $x_2 \geq 0$

Illustration-11

Find the Dual program of the following LPP –
Maximize $Z = 3x_1 + 5x_2 + 7x_3$ subject to $x_1 + x_2 + 3x_3 \leq 10$, $4x_1 - x_2 + 2x_3 \geq 15$ and x_1 , $x_2 \geq 0$ & x_3 is unrestrictedin sign.

Illustration-12

A retired person has plans to invest in shares. He has been suggested by one of his friends who plays in the share market to invest in two shares A and B which gives dividends @ 12% and 4% p.a respectively. For an investment of `1, the growth in the market value of the shares A and B are respectively 10 paise and 40 paise in one year. The retired person wants to invest such that the dividend income is at least `600 p.a and the growth of initial investmentin one year is at least `1000.

(i) Formulate it as a Linear Programming Problem.
(ii) Write its Dual.
(iii) Solve the Dual using Simplex Method. Interpret the solution.

Multiple Choice Questions

1. A constraint in an L.P. Model restricts
   a. Value of the Objective Function.
   b. Values of the Decision Variables
   C. Use of the available resources
   d. All the above
2. In graphical method of solution of LPP if the Iso-cost line coincide with a side of the Feasible

Region thenwe get –

a. Unique optimum solution.
b. Unbounded optimum solution.
c. No feasible solution.
d. Infinite number of optimum solutions.

3. A feasible solution of LPP –
   a. Must satisfy all the constraints simultaneously.
   b. Need not satisfy all the constraints, only some of them.
   c. Must be a corner point of the feasible region
   d. All the above.
4. The Objective Function of a LPP is $Z = 3x_1 + 2x_2$. If $x_1 = 10$ and $x_2 = 5$ then the value of Z is –
   a. 35 b. 40 c. 45 d. 50
5. Multiple solution exist in a Linear Programming problem when –
   a. One of the constraints is redundant
   b. Objective Function is parallel to one of the constraints
   c. Two constraints are parallel
   d. All of the above
6. The linear function of the variables which is to be optimized is called –
   a. Constraints
   b. Objective Function
   c. Decision variables
   d. None of the above
7. If the value of the Objective Function can be increased or decreased indefinitely then the solution is called –
   a. Unbounded
   b. Bounded
   c. Infeasible
   d. None of the above
8. The first step in formulating a LPP is –
   a. Identify the upper and lower boundaries of the decision variables
   b. State the constraints as linear combinations of the decision variables
   c. Understand the problem
   d. Identify the Decision Variables
9. The best use of Linear Programming is to find the optimal use of –
   a. Manpower
   b. Material
   c. Money
   d. All of the above
10. Which of the following is assumption of Linear Programming Model?
   a. Divisibility
   b. Proportionality
   c. Additivity
   d. All of the above
11. Non-negativity condition of Linear Programming implies –
   a. A positive coefficient of variables in Objective Function.
   b. A positive coefficient of variables in any constraint.
   c. Non-negative value of resource.
   d. None of the above.
12. If the constraints of a Linear Programming problem are $x_1 + x_2 \leq 1$, $3x_1 + x_2 \geq 3$ and $x_1, x_2 \geq 0$ then –
   a. There are two feasible regions
   b. There are infinite feasible regions
   c. No feasible region
   d. None of the above

13. For any LPP the intermediate solutions must be checked by substituting them back into the
    a. Objective Function
    b. Constraints
    c. Either of (a) and (b)
    d. This is not required.
14. The feasible solution of any LPP should belong to –
    a. Both first and second quadrant
    b. Only first quadrant
    c. Only second quadrant
    d. Both first and third quadrant
15. The true statement related to the graphs of $3x_1 + 2x_2 \leq 5$ and $6x_1 + 4x_2 > 10$ is –
    a. Both the graphs are disjoint.
    b. Both contain the point (1,1) simultaneously
    C. Both (a) and (b) above are true
    d. Both (a) and (b) are not true
16. In which quadrant the bounded region of the inequalities $x_1 + x_2 \leq 1$ and $x_1 - x_2 \leq 1$ is situated?
    a. First and third
    b. Second and third
    c. First and second
    d. All the four quadrants
17. Objective function of LPP is –
    a. A relation between the variables
    b. A function to be optimized
    c. A constraint
    d. None of the above
18. The optimal value of the Objective Function is attained at the poinrs
    a. Given by intersection of inequations with axes only
    b. Given by intersection of inequations with x axis only
    c. Given by intersection of inequations with y axis only
    d. Given by corner points of the feasible region.
19. If the constraints in a Linear Programming problem are changed then –
    a. The problem is to be re-evaluated.
    b. Solution is not defined
    C. The Objective Function has to be modified.
    d. The change in constraints is to be ignored.
20. The constraints $y - x \leq 1$, $3y - x \leq 9$ and $x, y \geq 0$ are defined on
    a. Bounded feasible space
    b. Unbounded feasible space
    c. Redundant space
    d. None of the above.
21. Which of the terms is not used in Linear Programming?
    a. Slack variables
    b. Objective function
    c. Concave region
    d. Feasible region
22. The area of the Feasible Region of the constraints $3x_1 + x_2 \geq 3$, $x_1 \geq 0$ and $x_2 \geq 0$ is –
    a. Bounded
    b. Unbounded
    c. Convex
    d. Concave
23. For the LPP, Minimize $Z = x + y$ subject to the constraints $5x + 10y \leq 0$, $x + y \geq 1$, $y \leq 4$, $x \geq 0$ and $y \geq 0$
    a. There is a bounded solution
    b. There is no solution
    c. There are infinite solutions
    d. None of the above
24. In a Linear Programming Problem –
    a. Objective Function is linear.
    b. Constraints are linear.
    c. Both Objective Function and Constraints are linear.
    d. None of the above

25. Constraints mean –
    a. Limitations are expressed in the form of mathematical inequalities or equalities.
    b. Assumption
    c. Goal to be achieved
    d. None of the above
26. The region which satisfies all the constraints of LPP is known as –
    a. Phisible region
    b. Convex region
    c. Feasible region
    d. Concave region
27. In LPP while drawing the graph, y values on x axis are always –
    a. 1 b. 0 c. -1 d. All of the above
28. The set of decision variables which satisfies all the constraints of LPP is called –
    a. Solution
    b. Basic solution
    c. Feasible solution
    d. None of the above
29. The value of the Objective Function is maximum under linear constraints –
    a. At the centre of Feasible Region
    b. At the origin
    c. At a vertex of the Feasible Region
    d. .At the vertex of the Feasible Region which is farthest from the origin.
30. A solution which optimizes the Objective Function is called –
    a. Solution
    b. Basic solution
    c. Feasible solution
    d. Optimal Solution

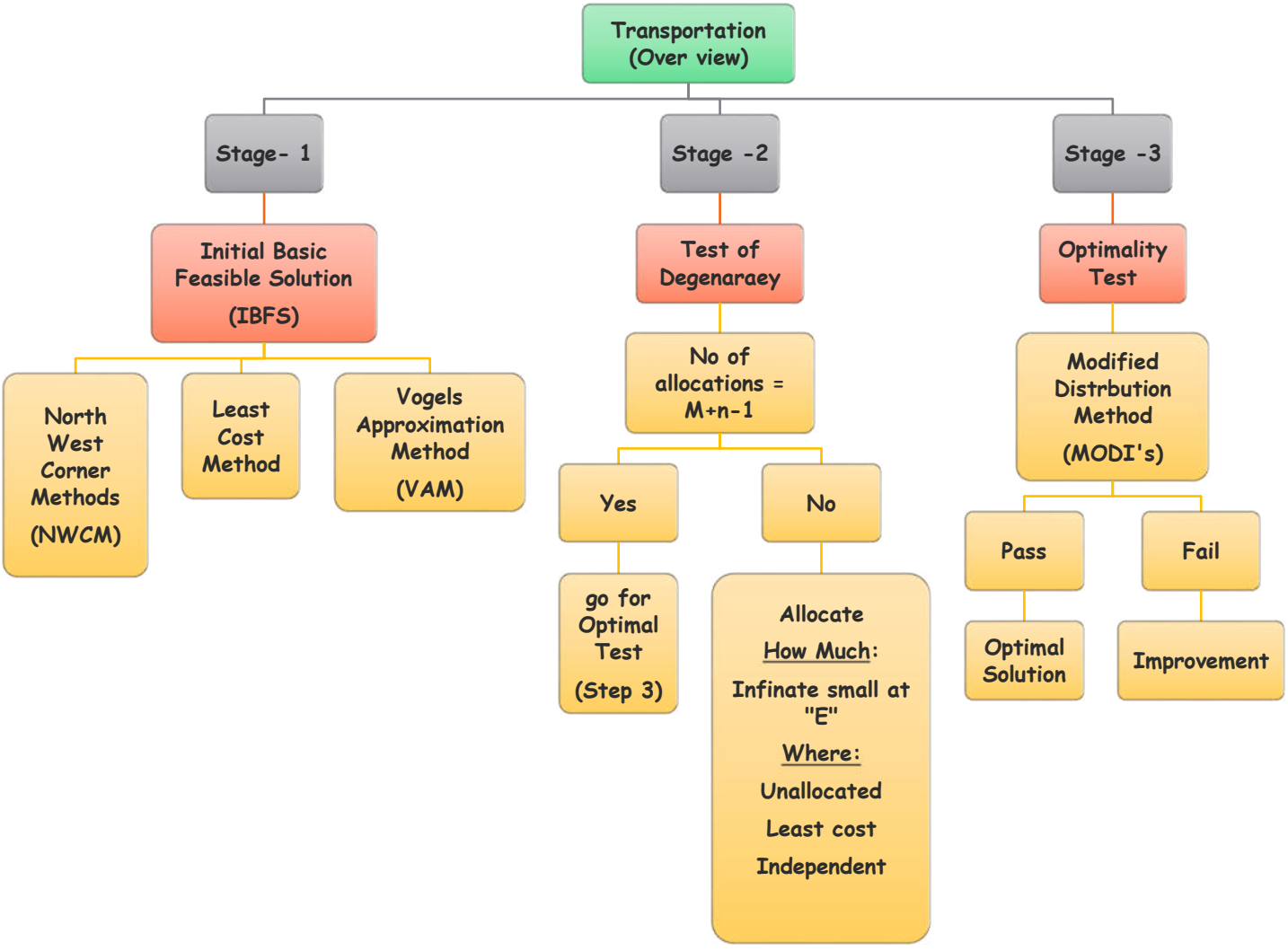
Answers

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
c	b	a	b	b	b	a	d	d	d	c	c	d	b	a
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
d	b	d	a	b	c	b	a	d	a	c	b	c	d	d

7. Transportation

Introduction

1) Features:
   a) A type of linear programming problem
   b) It is all about matching demand and supply
   c) No need to have one to one relationship i.e., a factory can supply to multiple warehouses or awarehouse can take from multiple factories.
   d) Like assignment transportation steps also can solve only minimization types
2) **Stages** in solving a transportation problem:



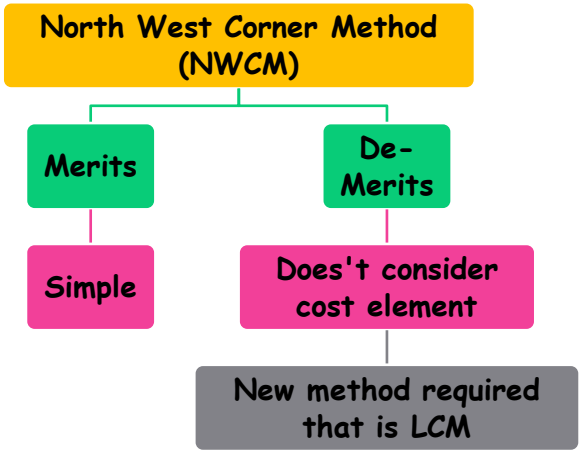
STAGE -1 IBFS

North West Corner Method (NWCM)

While Solving the NWCM probles following steps need to be followed

1. Idenitify the least cost element across the Matrix
2. Allocate the maximum quantity at this least cost cell and cancle the row/columu in which supply/Demand exhaust
3. Identify the next least cost element and allocate at that cell
4. Repeat 1-3 steps untile the total supply/demand exhast

Note Points



Least cost method (LCM):

Step 1:

(a) Select the cell with the lowest transportation cost among all the rows or columns of the transportationtable.

(b) If the minimum cost is not unique, then select arbitrarily any cell with this minimum cost.

Step 2:

Allocate as many units as possible to the cell determined in step 1 and eliminate that row (column) in which either supply is exhausted or demand is satisfied.

Step 3:

Repeat steps 1 and 2 for the reduced table until the entire supply at different factories is exhausted to satisfythe demand at different warehouses

IBFS under VAM

Vogel's Approximation Method (VAM)

Step 1: Calculation of Row and Column Penalties

a. Identify the least element and next least cost element in each row and column
b. Write the difference between the two least costs
c. This difference is called penalties

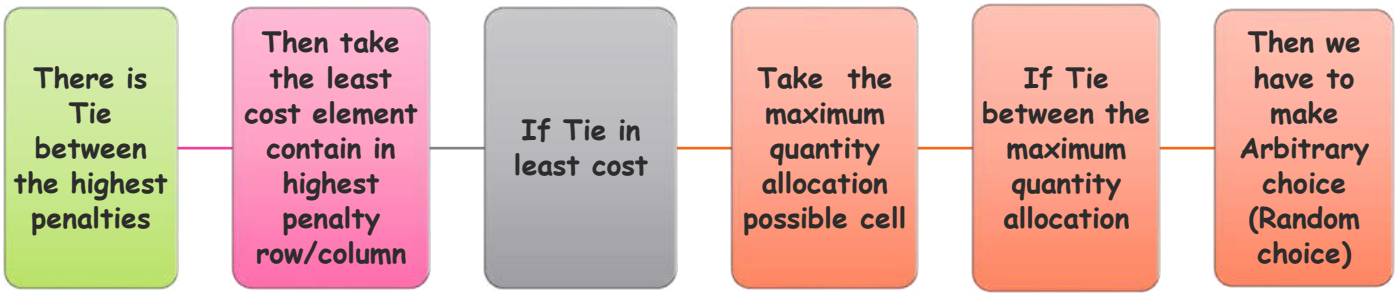
Step 2: Identify the highest penalty row OR column

a. In the highest penalty row/ column identify the least cost element
b. Make an allocation at that least cost cell, by looking at supply and demand
c. Cancel the row /& column at which the supply and demand exhausted

Step 3: Repeat the Step 1 and 2 until the total demand and supply exhausted

Note Points:

In step-2.a



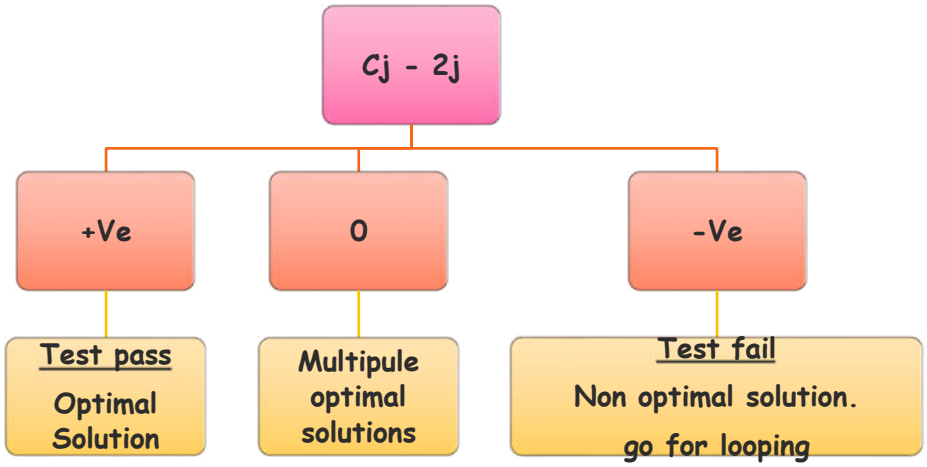
STAGE -2 TEST OF DEGENARACY

STAGE-3 OPTIMALITY TEST (MODIs)

In stage -3 MODIs method: Cj-Zj

1) How to understand $C_j - Z_j$?
   a. If '$C_j - Z_j$' value of the unallocated cell is '6'. This means if this cell is made allocated for every allocation made the cost increases by Rs.6.

b. If there exists any negative number improvement is made because there exists scope forreduction of cost.



Types of problems in Transportation:

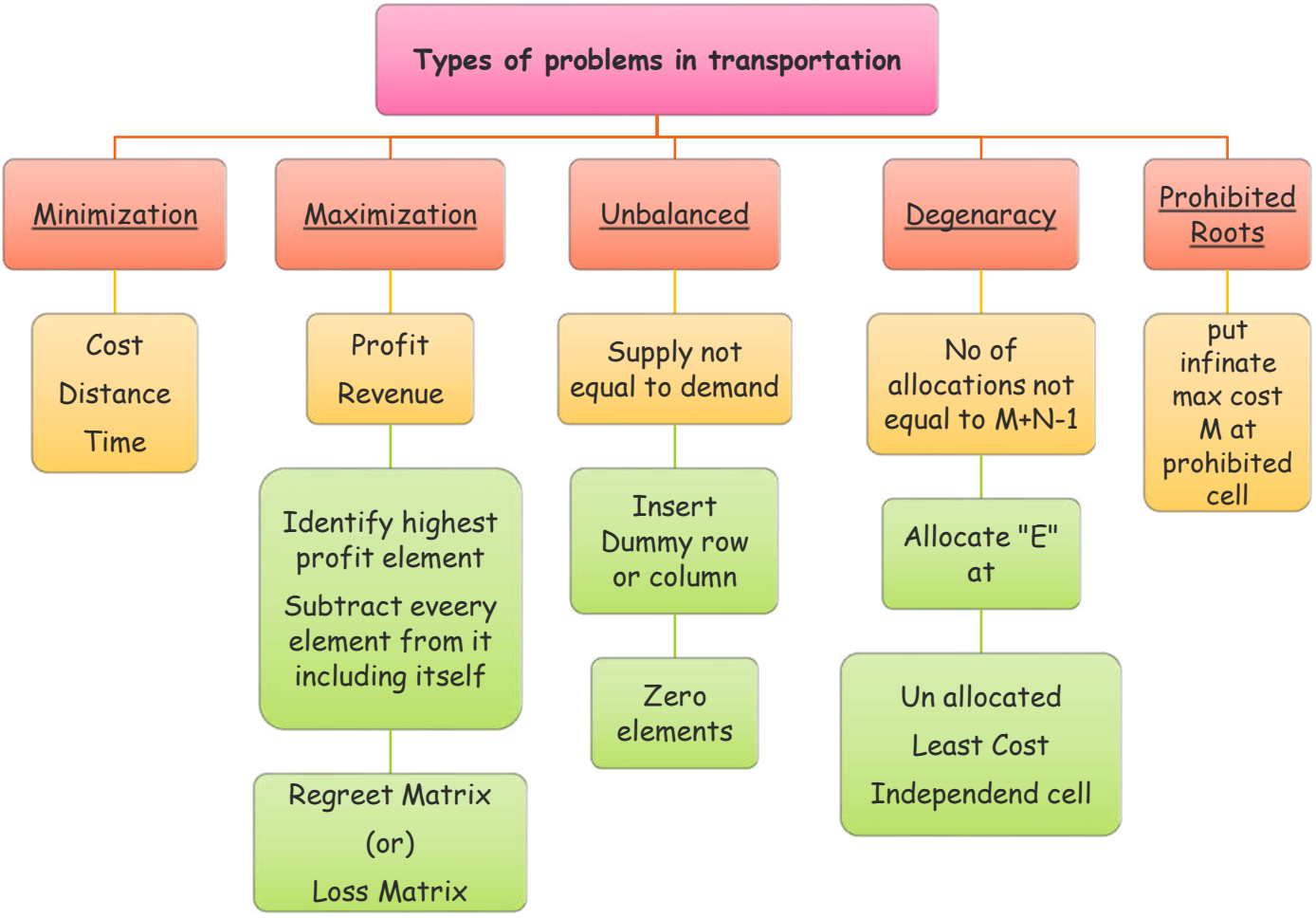


Illustration-1 (NWCM)

A company dealing with a special type of liquid has three plants P_1, P_2 & P_3 located throughout the country. Production capacities of these plants are respectively 50, 75 and 25 Gallons. Each day the firm must furnish to four of its Retail shops R_1, R_2, R_3, and R_4 with at least 20, 20, 50 and 60 Gallons of the product. The transportation cost per Gallon (in '000 *Rs*) between various sets of Plants and Retail Shops are given below –

From Plants	to Retail Shops				Supply
	R_1	R_2	R_3	R_4	
P_1	3	5	7	6	50
P_2	2	5	8	2	75
P_3	3	6	9	2	25
Demand	20	20	50	60	

Find the Basic Feasible Solution for the problem of transportation using NWCM.

Illustration-2 (LCM.)

Use data of Illustration 1, to find the Basic Feasible Solution by LCM.

Illustration-3 (VAM)

Use data of Illustration 1, to find the Basic Feasible Solution by *C*

Illustration-4 (MODI)

Test the Basic Feasible Solution obtained by VAM in the previous Illustration for Optimality. In case the solution is non-optimal, find the optimum solution to get the minimum Total Cost of Transportation.
KEY POINTS

Illustration-5 (UN BALANCED)

The products of two Plants A and B are to be transported to three Warehouses W_1, W_2 and W_3. The costs (*Rs* '00)of transportation of each unit from Plants to the Warehouses are indicated in the table below. Also provided are the Supply Capacities of the Plants and the Demands of the three Warehouses.

	Warehouse W_1	Warehouse W_2	Warehouse W_3	Supply Capacity
Plant A	25	17	25	300
Plant B	15	10	18	500
Demand	300	300	500	1100 \ 800

Find the Optimum Distribution Schedule and associated Cost of Transportation.

Illustration-6 (MAXIMISATION -PROBLEM)

A multi plant company has three manufacturing plants M_1, M_2 and M_3. The company is dealing with a unique product and enjoy monopoly as far as competition is concerned. They have two fixed customers A and B who procures all the items produced by the company.

Cost of Production (*Rs* per piece of the product) of the Pants M_1, M_2 and M_3 are respectively 1500, 1600 & 1700. Selling prices to the customers A and B are *Rs* 4400 and *Rs* 4700 per piece respectively.

Production Capacities of the three Plants, Demands of the two Customers and the Costs of Transportation per unit from various Plants to the different Customer's premises are given as follows:

	Plant M_1	**Plant M_2**	**Plant M_3**	**Demand**
Customer A	*Rs*1000	*Rs*2000	*Rs*1500	3500 units
Customer B	*Rs*1500	*Rs*3000	*Rs*2500	3600 units
Production Capacity	2000 units	3000 units	4000 units	

Formulate the problem as LP Model. Also find the optimum solution using Transportation algorithm.

KEY POINTS

Illustration-7 (PROBLEM WITH DEGENARACY)

A manufacturing company has three Pants X, Y and Z which supply to the Distributors located at A, B, C, D and

A. Monthly production capacities of the Plants are respectively 80, 50 and 90 units. Monthly requirement of the Distributors are 40, 40, 50, 40 and 80 units respectively. Unit Transportation Costs (*Rs*) are given below.

from Plants	**to Distributors**				
	A	**B**	**C**	**D**	**E**
X	5	8	6	6	3
Y	4	7	7	6	6
Z	8	4	6	6	3

Determine the Optimum Schedule of distribution of the company in order to minimize the Total Cost of Transportation.

KEY POINTS

Illustration-8

Find an Alternative Solution to the Transportation Problem of Illustration 5.

KEY POINTS

IMPORTANT NOTES

1) Suppose there is a tie in the negative cells quantities. For example, instead of 10 and 20 we have 10 and 10, then by making new allocation two existing old allocation will be replaced against degeneracy will arise. Resolve the degeneracy before proceeding for the optimality test for the improved solution. Thus degeneracy may occur in two stages:
   i. During IBFS (Initial Basic Feasible Solution) – Vogel's Approximation
   ii. During Moody's improvement
2) What happens if you are not able to form a loop during improvement?
   Answer: Such situation will never arise because the procedures ensure that all unallocated cells during optimality test are depended cells. That is reason why during degeneracy we make 'e' allocation in LEAST COST INDEPENDEDN UNALLOCATED CELL. So that no independent unallocated cellremains.
3) If one of the negative cell has 'e' quantity the reshuffling will change the allocations but will not change the cost, should we do the reshuffling?
   Answer: Yes, because the rearranged matrix will change U_iV_j values highlighting more possible improvements
4) Can we make 'e' allocation in dummy cell?
   Answer: Yes
5) Will there be two 'e' allocations?
   Answer: Yes, it is possible

Multiple Choice Questions

1. Which of the following considers difference between least cost and the cost just before least for each rowand column while finding Basic Feasible Solution in Transportation?
   (a) North West Corner Method (c) Vogel's Approximation Method
   (b) Least Cost Method (d) Both (b) and (c) above
2. When the total allocation of a Transportation Problem match with supply and demand values, the solution is –
   (a) Non-degenerate (c) Degenerate
   (b) Feasible (d) None of the above
3. The solution to a Transportation Problem with 'm' sources ad 'n' destinations is feasible if the number of cell allocations are –
   (a) m + n (c) m – n -1
   (b) mn (d) m + n – 1
4. To resolve Degeneracy in the solution of a Transportation Problem an infinitely small allocation is made to the solution already obtained. This allocation is known as-
   (a) Dummy (c) ε – the Greek letter
   (b) Epsilon (d) All of the above except (a)
5. Which of the following is not correct with respect to Transportation as a tool of Quantitative Technique?
   (a) Transportation technique is a special case of LP.
   (b) Transportation technique might give rise to solutions which are degenerate.
   (c) No Transportation problem can be given with supply ≠ demand.
   (d) Using Transportation technique one can maximize an Objective Function.
6. Which of the following method is used to test optimality of a solution in Transportation?
   (a) Modified Distribution (c) VAM
   (b) Simplex (d) LCM
7. In a solution of Transportation problem, empty cells are called –
   (a) Unoccupied cells (c) Empty cells
   (b) Unallocated cells (d) All of the above
8. The Transportation Problem deals with the transportation of –
   (a) Single product from a source to several destinations
   (b) Several products from a source to a destination.
   (c) Single product from several sources to a destination.
   (d) Single product from several sources to several destinations.
9. In NWCM, first allocation is made at –
   (a) Upper left hand corner of the table.
   (b) Lower right hand corner of the table.
   (c) Upper right hand corner of the table.
   (d) Lower left hand corner of the table.
10. One of the disadvantages of North West Corner rule for finding Initial Feasible Solution of Transportation problem is –

(a) It is complicated to use
(b) It leads to non-optimal solution
(c) It does not take into account unit cost of transportation.
(d) Generally it provides degenerate solution.

11. When total demand and supply are equal then the Transportation problem is said to be –
(a) A problem having multiple optimum solutions.
(b) A problem having degeneracy.
(c) A balanced one.
(d) None of the above.

12. Which one of the following is correct?
(a) The dummy source or destination is used in a Transportation problem to resolve degeneracy.
(b) The dummy source or destination is used in a Transportation problem to make it balanced.
(c) The dummy source or destination is used in a Transportation problem to ensure its cost effectiveness.
(d) All the above statements are correct.

13. For solving a maximization problem by Transportation algorithm, the very first step is to –
(a) Subtract smallest cost element of the matrix from all the other cost elements.
(b) Subtract all the cost elements of the matrix from the highest element of the same.
(c) Add smallest cost element of the matrix to all the other cost elements.
(d) Add highest cost element of the matrix to all the other elements.

14. Which of the following methods is used for finding an initial feasible solution of a Transportation Problem?
(a) Simplex
(b) Least Cost
(c) Hungarian
(d) Big M

15. Which of the following is a method for improving an initial solution of a Transportation problem?
(a) Stepping Stone
(b) North West Corner
(c) Intuitive Lowest Cost
(d) All of the above

16. Basic Feasible Solution for a Transportation problem is given as follows –

To	Warehouse		Supply
From	W_1	W_2	
A	5 (10)	7	10
B	8 (15)	9 25	40
Demand	25	25	50

Given, the Unit Transportation Costs are in Rupees. Can this solution be improved?
(a) Yes the solution can be improved by Rs 50

(b) Yes the solution can be improved by *Rs* 100
(c) No the solution is optimal
(d) Yes the solution can be improved by *Rs* 10.

17. The Initial Feasible Solution of a Transportation Problem can be obtained by different methods. The only restriction is that –
(a) The edge constraints of supply and demand are satisfied.
(b) The solution must be obtained using VAM.
(c) The solution should be non-degenerate.
(d) All of the above.

18. The purpose of Stepping Stone Method is to –
(a) Facilitate moving from a feasible solution to an optimal solution.
(b) Test optimality of a solution.
(c) Both the two above.
(d) None of the above.

19. Which one of the following is the purpose of a dummy source or dummy destination in a Transportation Problem?
(a) To convert the problem from unbalanced to balanced.
(b) To make the solution non-degenerate.
(c) To provide a means of a dummy problem.
(d) To make sure that the total cost is not exceeding a predetermined figure.

20. An important assumption of Transportation technique is –
(a) There is only one optimal solution for each problem
(b) There are no economies of scale if huge quantities are transported from one source to one destination.
(c) The number of dummy sources and destinations must be equal.
(d) None of the above.

21. The equation $C_{ij} = u_i + v_j$ is used to calculate –
(a) An improvement index for the Stepping Stone Method.
(b) The MODI cost values u_i and v_j
(c) The Degeneracy index.
(d) None of the above

22. For an unbalanced problem of Transportation, the cost coefficients for each of the created cells is –
(a) Very high positive value
(b) Very high negative value
(c) Zero
(d) One

23. A degenerate solution of a Transportation Problem means –
(a) Total supply is not equal to the total demand.
(b) Some allocations have become negative.
(c) The obtained solution is not feasible.
(d) Both (a) and (b) but not (c).

24. Multiple optimum solutions exist for a Transportation Problem when –
(a) There is at least one unoccupied cell of the obtained optimal solution which has zero

opportunity cost.

(b) There is unused route of Transportation having all the cells with positive opportunity cost.

(c) There is unused route of Transportation with further scope of reducing total cost of transportation.

(d) There is one and only one unoccupied cell of the obtained optimal solution with zero opportunity cost.

25. In an iteration while moving from one solution to the next, degeneracy occurs when –

(a) The closed loop indicates a diagonal move.

(b) Two or more of the allocated cells in the closed loop with minus sign have same lowest allocation.

(c) Two or more allocated cells are on the closed loop but neither of them represent a corner of the loop.

(d) Either one of the above

26. Left hand side of the equation $\Delta_{ij} = C_{ij} - (u_i + v_j)$ is known as –

(a) Opportunity Cost

(b) Improvement Index

(c) Both (a) and (b)

(d) None of the above

27. The highest negative opportunity cost value in an unused cell of a Transportation Matrix is chosen to improve the current solution because –

(a) It represents maximum possible cost reduction per unit

(b) It ensures no violation of Rim Condition.

(c) It represents per unit cost improvement.

(d) Either one of the above.

28. Which of the following statements is best suited to the Transportation solution given below?

To / From	Warehouse W_1	Warehouse W_2	Warehouse W_3	Supply
A	3 **20**	5 **50**	9	70
B	5	4 **30**	7	30
C	10 **40**	8	3 **80**	120
Demand	60	80	80	220

(a) The solution is degenerate.

(b) The solution can be improved by shipping from C to W_2

(c) The solution can be improved by shipping from B to W_1

(d) NWCM has been used to develop the solution.

29. The Total Cost of Transportation for the Solution Matrix given in the Q. No. 28 is –

(a) Rs1070

(c) Rs1350

(b) Rs1130 (d) Rs1050

30. Which of the following statement is true in respect of the solution of a Transportation Problem?

To / From	Warehouse W_1	Warehouse W_2	Warehouse W_3	Supply
A	25	17 300	25	300
B	15 300	10	18 200	500
C	0	0	0 300	300
Demand	300	300	500	1100

(a) The problem is an unbalanced one with Demand > Supply.

(b) Plant C is a Dummy Plant.

(c) Demand of Warehouse W_3 will not be completely fulfilled.

(d) All of the above.

Answers:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
c	b	d	d	c	a	d	d	a	c	c	b	b	b	a
16	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
c	a	c	a	b	b	c	c	a	b	c	a	b	a	d

8. Assignment

1) Features:

a) A type of linear programming problem
b) Assigning jobs to men & women
c) Can solve only minimization types
d) The assignments pre-condition is one to one relationship.
e) Assignment problem will be solved by using HUNGARIAN METHOD

2) Types of Problems

a) Minimization balanced assignment problems
b) Maximization balanced assignment problems
c) Minimization unbalanced assignment problems
d) Maximization unbalanced assignment problems
e) Degeneracy
f) Prohibited routes
g) Multiple optimal solution
h) Travelling sales men problem (Concept of looping)

3) Types of LPP

a) Linear Programing Problems
   i. Normal Linear Programming Problems
   ii. Transportation problems
      1. Normal transportation problems
      2. Assignment problems

Hungarian method:

Check-1: Objective check :?

check whether problem is of minimization problem or maximization problem.
If minimization- proceed to next check
If maximization – convert matrix into loss matrix or regrette matrix

Check-2: Check whether balanced or not:?

Check whether problem is balanced-i.e. no.of rows = no.of columns ?.
If not insert column or row (as per requirement) with 0"s as elements.

Steps involved in the minimization of an assignment problem under Hungarian method:

Step 1: Row Operation

Locate the smallest cost element in each row of the given cost table.
Now subtract this smallest element from each element in that row.
result: there shall be at least one zero in each row of this new table, called the reduced cost

table.

Step 2: Column Operation

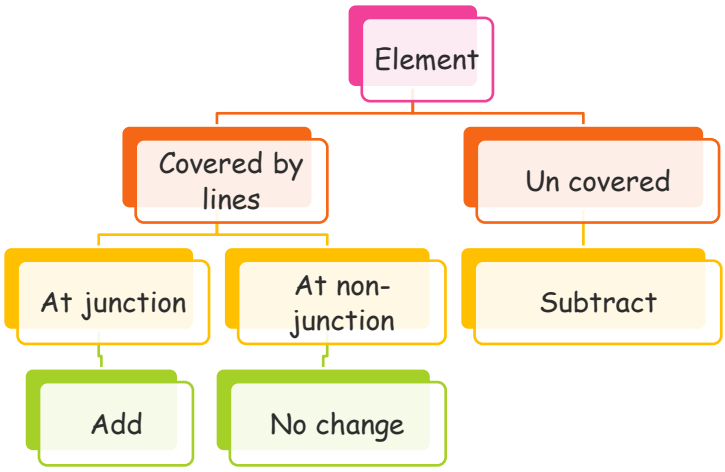
In the reduced cost table obtained, consider each column.
locate the smallest element in each column.
Subtract the smallest value from every entry in the column,
Result: There would be at least one zero ineach of the rows and columns of the second reduced cost table.

Step 3: Optimality test:

Draw the minimum no. of horizontal and vertical lines (not the diagonal ones) that are required to cover all the zero elements. (tip:find highest 0s column or row draw the line- repeat) .
Whether the no. of lines drawn is = 'n' ? (n=order of matrix i.e the no. of rows/columns of the given Cost Matrix) .
Yes ; the solution is optimal and proceed to step 6.
No : solution is non optimal step 4.

Step 4: Improved Matrix:

Select the smallest uncovered (by the lines) cost element.
Subtract this element from all uncovered elements including itself.
Add this element to each value located at the intersection of any two lines.
The cost elements through which only one line passes remain unaltered.



Step 5: Repeat step 3 and 4 until an optimal solution is obtained.

Step 6: Optimal allocations:

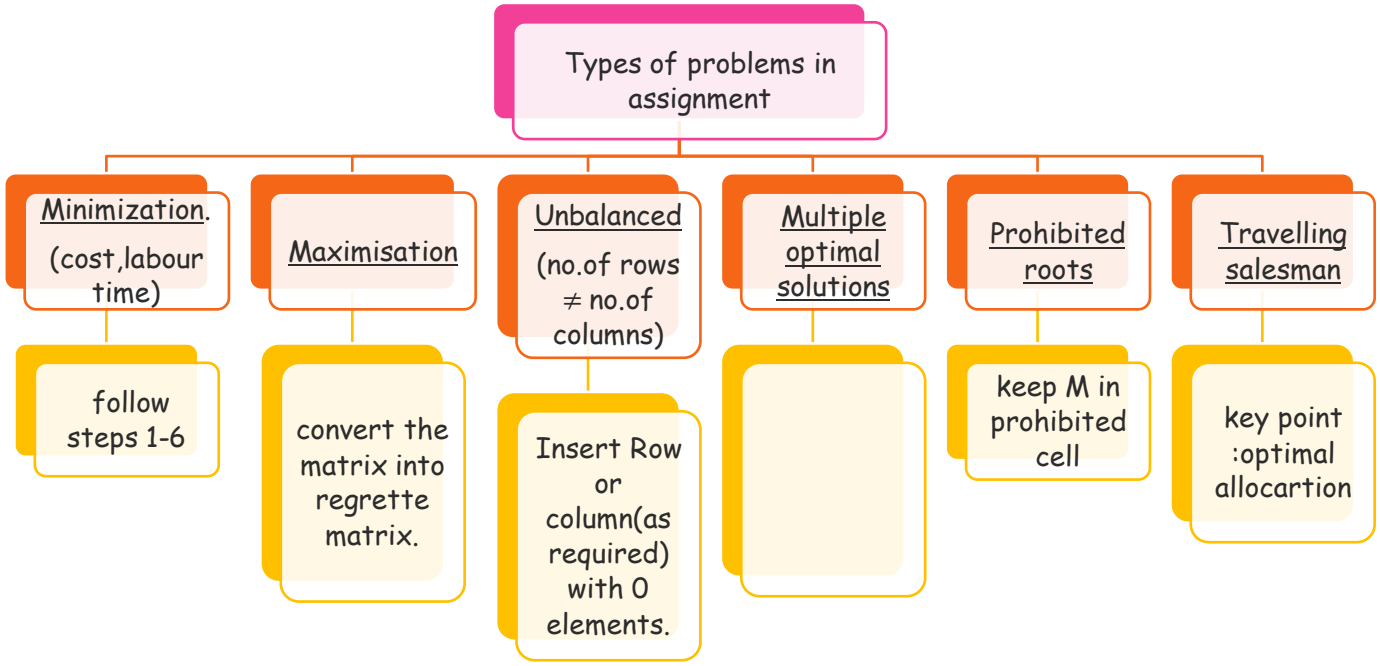
Given the optimal solution, make the job assignments as indicated by the 'zero' elements. This is done as follows:

(a) Locate a row which contains only one zero element. Assign the job corresponding to this element to its corresponding person.
Cross out the zero's if any in the column corresponding to the element, which is indicative of the fact that the particular job and person are no more available.

(b) Repeat (a) for each of such rows which contain only one zero. Similarly,

perform the same operation in respect of each column containing only one 'zero' element, crossing out the zero(s), if any, in the row in which the elements lies.

(c) If there is no row or column with only a single 'zero' element left, then select a row/column arbitrarily and choose one of the jobs (or persons) and make the assignment. Thus in such a case, alternative solutions exist.



Maximization model:

(How to identify maximization model? – profit matrix, revenue matrix, etc.)

- Convert profit matrix to relative loss matrix/(regret matrix).

  This can be done by taking the highest number in the matrix and reducing all the other numbers from it.Due to this we convert the matrix into regret matrix and by minimizing regret we can maximize production

- Now use Hungarian method to minimize loss matrix.
- Once assignments are made . find the total value of profit.

Unbalanced assignment:

- If no of rows ≠ no.of columns – problem is unbalanced
- How to balance?
  Ans: Insert additional row or column (as the case may be)
- In fact theintroduction of additional row or column is needed to make the matrix square which is the prime requirement of anAssignment Problem.
- This newly introduced row or column is called Dummy Row or Dummy Column.

Assignment Problem with Multiple Optimum Solution or Alternative Solution

- If while assigning it is seen that neither any row nor any column is having single zero.
- In such case one has to firstly check the rows and then columns to find one with two zeros.
- One of these should be chosen arbitrarily and a square to be drawn around it to mark an assignment at that cell.
- The other zero in the samerow or column should be crossed out.
- Alternatively the cell whose zero is bounded by drawing a square aroundit should be crossed out and the other one is assigned. This will lead to two Alternative Solutions
- For situations with more than two zeros, the procedure is similar i.e.. one zero to be chosen arbitrarily and the others should becrossed out.

Problems with Restriction on Assignments or Prohibited Assignment

- when a particular resource (say, a person) cannotbe assigned to a particular activity (say, a job).
- To handle such a problem a very high cost (or time which is to be minimized) is assigned to that cell of the matrix which is meant for this restricted or prohibited assignment.
- This automatically restricts any assignment at that cell. Very high cost is generally represented by ∞ or M.

Note: If in spite of assigning 'M' to the prohibited cell, if allocation is made then the problem is said to be having infeasible solution

Problem of Travelling Salesman

- This can be considered as a special case of Prohibited Assignment.
- Here a Salesman has to start his sales tour from a particular city and visit all the other cities within in such a manner that he ends his tour at the same city from where he started.
- Objective of the salesman is to schedule the tour in such a way that thetotal distance travelled or total time elapsed for the travel or total cost of travel is minimised.
- This type of problemscan be solved by the algorithm used for Assignment.

Illustration-2 (MINIMISATION)

An equipment under breakdown has five repair jobs to make it operative again. The Maintenance Manager of theorganisation has assigned five mechanics of his department to do the jobs. The estimated time (hours) for each ofthe mechanics to carry out the jobs are given in the following table.

	Time required (Hours) to complete the Repair jobs				
Mechanic	A	B	C	D	E
I	7	5	9	8	11
II	9	12	7	11	10
III	8	5	4	6	9
IV	7	3	6	9	5
V	4	6	7	5	11

Assuming that each mechanic can be assigned to only one job, determine the minimum time assignment.

Key point:

Illustration-3 MAXIMISATION

Five Salesmen are to be assigned to five Districts. Estimates of Sales Revenue (in Rs000 Rs) for each Salesman aregiven in the table below.

Districts	Salesman A	Salesman B	Salesman C	Salesman D	Salesman E
1	32	38	40	28	40
2	40	24	28	21	36
3	41	27	33	30	37
4	22	38	41	36	36
5	29	33	40	35	39

Find the assignment pattern that maximises Revenue.

Key point:

Illustration-4 UNBALANCED

A city corporation has decided to carry out road repairs on four main arteries of the city. The government has agreed to make a special grant of Rs 50 lakhs towards the cost with a condition that the repairs must be done at thelowest cost and quickest time. If conditions warrant, then a supplementary token grant will also be considered favourably. The corporation has floated tender and five contractors participated in

bidding. In order to expedite work, one road will be awarded to one contractor. The following matrix of Cost of Repairs is prepared by the corporation on the basis of the bids submitted by the participants.

	Cost of Repairs in Rs Lakhs for			
Contractors	**Road 1**	**Road 2**	**Road 3**	**Road 4**
A	9	14	19	15
B	7	17	20	19
C	9	18	21	18
D	10	12	18	19
E	10	15	21	16

(i) Find the best way of assigning the repair work to the contractors and the total cost.

(ii) If it is necessary to seek supplementary grant then what should be the amount sought?

(iii) Which of the five contractors will be unsuccessful in his bid?.

Key point:

Illustration-5 (MULTIPLE OPTIMAL SOLUTIONS)

Use the data of Illustration 3 above to find all the possible optimum solutions.

Key point:

Illustration-6 PROHIBITED ASSIGNMENT/ RESTRICTIONs

A company has taken on rent three floors (1st, 2nd and 3rd) of a multi storied building for their city office. It has been decided to locate Managers of Marketing, Purchase, HR, Finance and Corporate Law in the office. The management has earmarked in different floors five rooms having numbers 103, 201, 205, 302 and 304 for the above mentioned Managers. But no particular room has been allotted for any particular Manager and rather they have given option to indicate their preference of rooms so that decision can be taken by the management using some scientific method and subsequently arrangement of sitting of the subordinates of various Managers can be made. Managerial preferences are provided in the table below with 1st preference appearing in the top for each and every Manager.

Preference of Rooms of different Managers				
Marketing Manager	**Purchase Manager**	**HR Manager**	**Finance Manager**	**Company Secretary**
302	302	103	302	201
103	304	201	205	302

304	205	304	304	304
	201	205	103	
		302		

It is evident that most of the Managers have not given preference for all the available rooms because they feel that all the rooms do not have the facility they are looking for. Assuming that the preferences can be quantified by numbers, find out which manager should be assigned with which room to minimise the preferential measure.

Key point:

Illustration-7 (TRAVELLING SALESMAN)

A travelling salesman has to visit five cities. He wishes to start from a particular city, visit each city once and then return to his starting point. The travelling cost (in Rs00 Rs) between any two cities is given in the table below

	To City				
From City	**A**	**B**	**C**	**D**	**E**
A	M	5	8	4	5
B	5	M	7	4	5
C	8	7	M	8	6
D	4	4	8	M	8
E	5	5	6	8	M

Find the cost minimising sequence of visit.

Key point:

ILLUSTRATION11: CONSTRUCTION of matrix & MAXIMISAATION (2019)

A company has four zones open and four salesmen available for assignment. The zones are not equally rich in their sales potentials. It is estimated that a typical salesman operating in each zone would bring in the followingannual sales:

Zone: A: 1,26,000: Zone B:1,05,000; Zone C: 84,000; Zone D: 63,000.

The four salesmen are also considered to differ in ability. It is estimated that working under the same condition

theiryearly sales would be proportionately as follows:

Salesman P:7; Salesman Q: 5; Salesman R:5; Salesman S:4. If the criterion is maximum expected total sales, the intuitive answer is to assign the best salesman to the richest zone, the next best to the second richest zone and so on. Verify this by the method of assignment.

Multiple Choice Questions

1. Which of the following methods is used to solve the Assignment problems?
   (a) Stepping Stone Method (c) North West Corner Method
   (b) Hungarian Method (d) Vogel's Approximation Method
2. Assignment of work to men and machines is known as
   (a) Scheduling (c) Balancing of Line
   (b) Loading (d) None of these
3. In an Assignment matrix of size (5 × 5), the total number of decision variables in the objective function is –
   (a) 10 (b) 5 (c) 25 (d) 15
4. An Assignment problem is solved to minimise the total time required to complete three jobs on three different machines such that each job is processed by exactly one machine and each machine processes exactly one job. The minimum total processing time is found to be 480 minutes. After a few days of operation, there hasbeen a change in the design of the second job. Due to this, the processing time of the second job is increased by 15 minutes in either of the machines. The revised minimum total processing time will be –
   (a) 495 minutes (c) 480 minutes
   (b) 465 minutes (d) None of these
5. Assignment problem can be considered as a particular case of -
   (a) Transportation problem (c) Queuing problem
   (b) Sequencing problem (d) All of these
6. Dummy row or column is added in an assignment problem –
   (a) To prevent a solution to become degenerate.
   (b) To reduce the total cost of assignment.
   (c) To increase the profit function.
   (d) To balance total activities and total resources
7. While solving an assignment problem, an activity is assigned to a resource with zero opportunity cost
   because objective is to –
   (a) Reduce total cost of assignment to zero. (c) Minimise total cost of assignment.
   (b) Reduce cost of that assignment to zero. (d) Maximise total cost of assignment.
8. In an assignment problem –
   (a) First activity is assigned to first resource
   (b) Any number of activities can be assigned to each resource.
   (c) It depends on how many resources are available.

(d) Only one activity be assigned to each resource.

9. An assignment problem can be viewed as a special case of transportation problem in which the capacity from each source is____and the demand at each destination is____.
   (a) Unlimited, unlimited (c) One, one
   (b) One, unlimited (d) Unlimited, one
10. In marking assignments which of the following should be preferred?
   (a) Only row having single zero (c) Column having more than one zero
   (b) Only column having single zero (d) Only row / column having single zero.
11. The assignment matrix is always a _____
   (a) Rectangular matrix (c) Square matrix
   (b) Identity matrix (d) None of these
12. Maximisation assignment problem is transformed into a minimisation problem by _____
   (a) Adding each entry of a column to the maximum value of that column
   (b) Subtracting each entry in a column from maximum value in that column.
   (c) Subtracting each entry of the table from the maximum value of the table.
   (d) Adding each entry of the table to the maximum value in the table.

13. The assignment problem will have alternative solutions when it has _____
   (a) At least one zero in any row or column (c) Two diagonal elements are zeros
   (b) All rows have two zeros. (d) None of the above.
14. In the Hungarian Method of solving Assignment problem, the row reduction is obtained by
   (a) Dividing each row by the elements of the row above it.
   (b) Subtracting the elements of the row from the elements of the row above it.
   (c) Subtracting the smallest element from all other elements of the row.
   (d) Subtracting all the elements of the row from the highest element in the matrix.
15. The horizontal and vertical lines drawn to cover all zeros of the total opportunity matrix for an optimal
   solution must be –
   (a) Equal to m x n, where m = No. of rows & n = No. of columns.
   (b) Equal to each other.
   (c) Equal to m + n, where m = No. of rows & n = No. of columns
   (d) Equal to the Order of the matrix.
16. In a problem of Travelling Salesman, the diagonal elements of the matrix from top left corner are all –
   (a) Zeros (c) Ones
   (b) Negative (d)Infinitely large
17. The similarity between Assignment Problem and Transportation Problem is –
   (a) Both are rectangular matrices
   (b) Both are square matrices
   (c) Both can be solved by graphical method
   (d) Both have objective function and non-negativity constraints.

18. When we try to solve the Assignment problem by Transportation algorithm the following difficulty arises.
    (a) There will be a tie while making allocations.
    (b) The problem will get alternate solution.
    (c) The problem degenerates and we have to use epsilon to solve degeneracy.
    (d) The Assignment problem cannot be solved by Transportation algorithm.
19. The following character dictates that the Assignment matrix is a square one.
    (a) The allocations in Assignment problem are one to one.
    (b) Because we find row opportunity cost matrix.
    (c) Because we find column opportunity cost matrix.
    (d) Because after making allocations, horizontal and vertical lines are to be drawn.

20. An Assignment problem is considered as a special case of Transportation problem because –
    (a) The number of rows is equal to the number of columns
    (b) All x_{ij} = 0 or 1
    (c) All rim conditions are equal to 1
    (d) All of these
21. An Assignment problem can be solved by –
    (a) Simplex method
    (b) Transportation method
    (c) Both (a) and (b)
    (d) Only (b) but not (a)
22. The Hungarian Method for solving an Assignment problem can also be used to solve –
    (a) Transportation problem
    (b) Travelling Salesman problem
    (c) Both (a) and (b)
    (d) Not (a) but (b)
23. A firm is required to procure three items I, II & III from three vendors V_1, V_2 & V_3 respectively. The quotedprices in Rupees are given in the table below. The management policy clearly states that each item should be procured from only one vendor and each vendor should supply only one item. The minimum total cost ofprocurement is –

	VENDORS		
ITEMS	**V_1**	**V_2**	**V_3**
I	110	120	130
II	115	140	140
III	125	145	165

    (a) Rs 375 (b) Rs 385 (c) Rs 390 (d) None of the above
24. In a machine shop four jobs need to be assigned to four machines. Each of the jobs is to be assigned to one machine only at a time. The time in minutes required to complete different jobs in different machines is givenin the table below.

	MACHINES			
JOBS	**I**	**II**	**III**	**IV**
A	15	13	14	17

B	11	12	15	13
C	13	12	10	11
D	15	17	14	16

In order to ensure that the total time to complete the jobs is minimum, the optimal assignment of the jobs is

(a) A to IV, B to II, C to III and D to I
(b) A to II, B to I, C to IV and D to III
(c) A to II, B to I, C to III and D to IV
(d) A to IV, B to II, C to I and D to III

25. If there are n jobs and n workers, there would be –
(a) n! solutions
(b) (n – 1)! Solutions
(c) (n!).n solutions
(d) n solutions

26. The Assignment problem
(a) Requires that only one activity be assigned to each resource
(b) Is a special case of Transportation problem
(c) Can be used to maximise the resources
(d) All of the above

27. To proceed with the MODI algorithm for solving an assignment problem, the number of dummy allocationsneed to be added are –
(a) N (b) n -1 (c) 2n (d) 2n – 1

28. An optimal solution of an assignment problem can be obtained only if –
(a) Each row and column has only one zero element
(b) Each row and column has at least one zero element
(c) Both the diagonals of the matrix have zero element
(d) None of the above

29. The procedure used to solve Assignment problems wherein one reduces the original assignment costs to a table of opportunity costs is called----------
(a) Stepping Stone Method
(b) Matrix Reduction
(c) MODI Method
(d) Northwest Reduction

30. When a maximisation assignment problem is converted to minimisation problem, the resultant matrix is called
(a) Cost matrix
(b) Profit matrix
(c) Regret matrix
(d) Dummy matrix

Answers

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
b	b	c	a	a	d	c	d	c	d	c	c	d	c	d
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
d	d	c	a	d	c	d	b	b	a	d	b	d	b	c

Game theory

- To analyze such conflicting situations, some special mathematical model called "**Game Theory**" is used. It was first developed, to solve problems in economics, by Hungarian born American mathematician John von Neumannand his Princeton University colleague Oskar Morgenstern, a German born American economist in the year1944.
- Game Theory may be defined as a type of Decision Making situation when two or more intelligent and rationalopponents are involved under conditions of conflict and competition.
- It is a type of Decision Theory in which one's choice of action is determined after taking into account all possible alternatives available to the opponent participating in the same game.
- Game Theory does not insist on how a game should be played but tells the procedure and principles by which action should be selected.
- 'Game' is defined as an activity between two or more participants according to a set of rules, at the end of which each participant either gets some benefit or suffers some loss.

Basic Terms.

A. **Player** – A participant is called a Player.

B. **Play** – A Play of the game is said to occur when each Player has chosen a course of action.

C. **2 Person Game** – If the number of Players in a Game is two then it is called 2 Person Game. The term Person refers to an individual or a group aiming at a particular objective.

D. **N Person Game** – If the number of Players in a Game is N (where N > 2) then it is called N Person Game.

E. **Zero Sum Game** – If the sum of **the amounts won by all winners is equal to that lost by all losers** then the game is called Zero Sum Game. In other words, sum of the gains and losses in such a game is zero..

Two Person Zero Sum Games are also called **rectangular game** because their payoff matrix is in the rectangular form.

F. **Non Zero-Sum Game** – If the **sum of the gains or losses in a game is not equal to zero** then it is called a Non Zero Sum Game.

G. **Strategy:** It is the predetermined rule by which a Player while playing decides the course of action from hisown list of courses of action. There are two types of strategies – Pure and Mixed.

H. **Pure Strategy** – If a Player knows **exactly what the other Player is going to do**, a deterministic situationis obtained. The objective is to maximize the **gain**

I. **Mixed Strategy** – If a Player is guessing , which activity is to be selected by the other,a probabilistic situation is obtained. The objective in this case is to maximize the **expected gain.**

J. **Payoff** – The **outcome of playing** the game is known as Payoff. It is the **quantitative measure** of satisfactiona Player gets at the end of each play.

K. **Payoff Matrix** – This is a tabular representation showing the outcomes or payoffs corresponding to differentstrategies of the participating Players. Since a Game involves at least two Players, the table referred above always forms a matrix with some rows (m, say) and columns (n, say).

Rules of a Payoff Matrix are –

- Rows denote the activities or courses of action available to Player A who is considered as the maximising player.
- Columns denote the activities or courses of action available to Player B who is the minimising player.

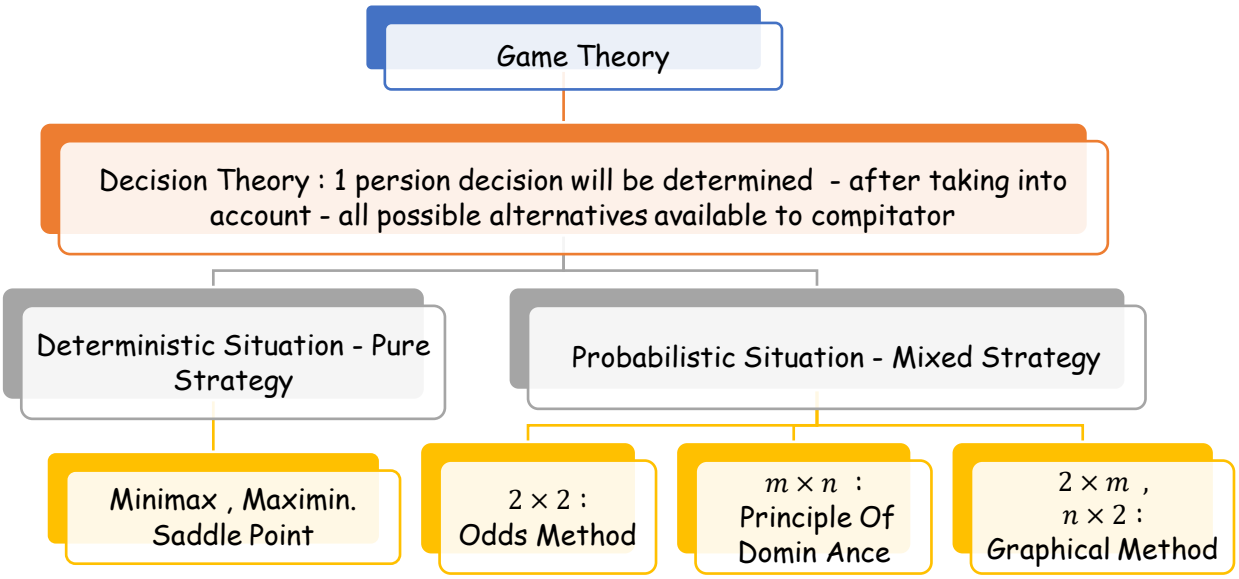
- The following table is an example of a Payoff Matrix of a Two Person Zero Sum Game which says that two firms are competing for business with the mentioned strategies so that one's gain is another's loss:

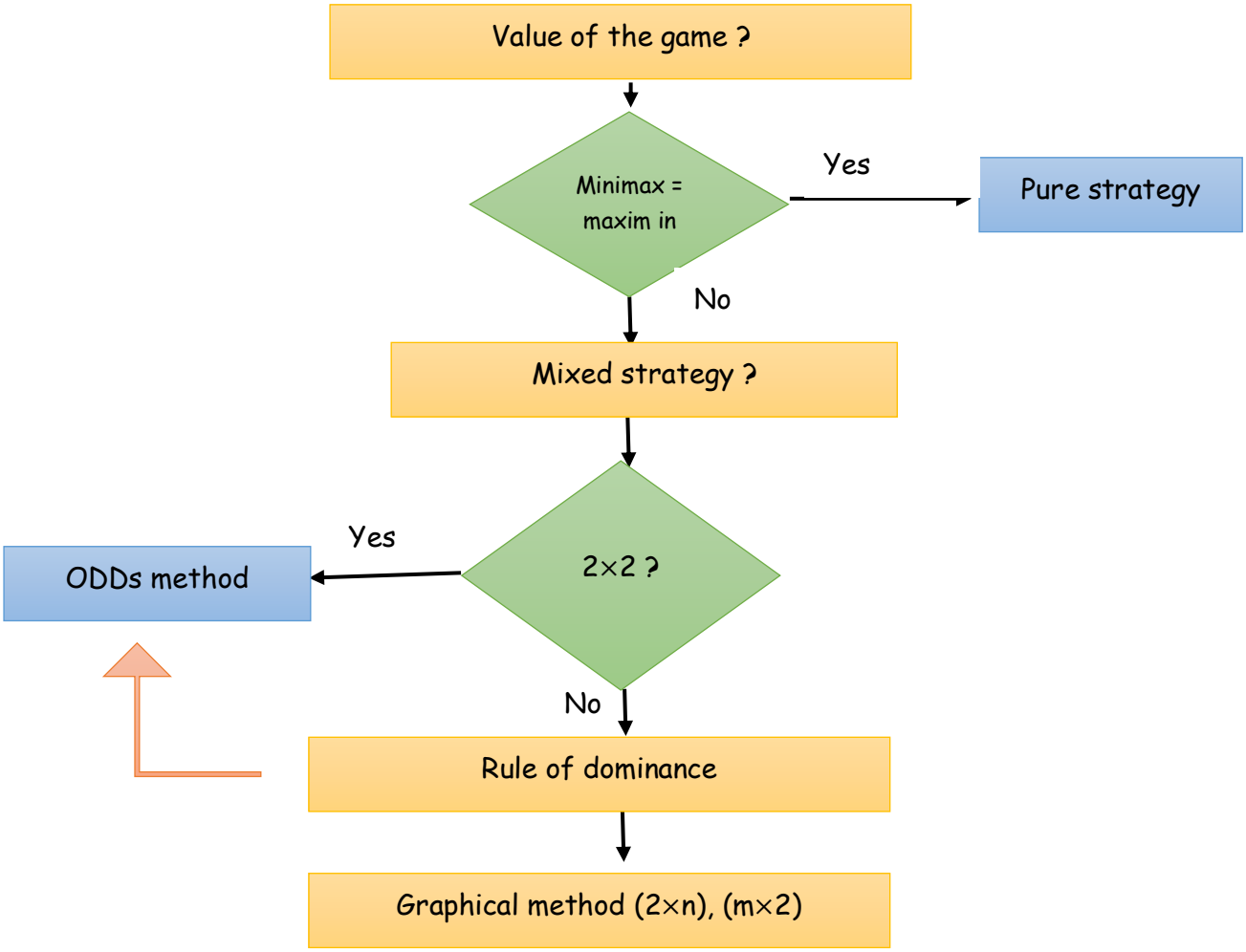
Strategies of Firm A	Strategies of Firm B		
	No advertising	Medium advertising	Heavy advertising
No advertising	10	5	-2
Heavy advertising	16	14	10

Here a positive payoff denotes gain to the maximising player i.e. Firm A (shown as Row) and loss to the minimising player i.e. Firm B (shown as Column). If Firm A chooses strategy "No advertising" and Firm B chooses strategy "Medium advertising" then gain of A will be 5 and loss of B will also be 5.

L. **Optimal Strategy:** A course of action which puts the Player in the **most preferred position**, irrespective of the strategy of his competitors, is called Optimal Strategy

M. **Value of the Game:** It is the **expected payoff** of play when all the players of the game follow their optimal strategies.
The game is called Fair if the **value of the game is zero** and **Unfair if it is non-zero**.





Solution of Pure Strategy Games with Saddle Point

- Pure Strategy Games are solved using Maximin – Minimax criteria.
- The maximising player (whose strategies are shown along the rows of the Payoff Matrix) arrives at his optimal strategy on the basis of Maximin criteria and
- theminimising player (whose strategies are shown along the columns of the Payoff Matrix) follows Minimax criteria.

The game is solved when Maximin and Minimax values are equal.

Maximin value is determined as follows –

(i) Find minimum value in each row of the given payoff matrix. This denotes minimum possible gain against each strategy of the Maximising Player.

(ii) Maximin value is the maximum of these minimum values.

Minimax value is determined as follows –

(i) Find maximum value in each column of the given payoff matrix. This denotes maximum possible loss against each strategy of the Minimising Player.

(ii) Minimax value is the minimum of these maximum values.

- Saddle Point is said to exist when the Maximin and Minimax values are equal.

- If there is more than one Saddle Point then more than one solution will be possible corresponding to each Saddle Point.

Solution of Mixed Strategy Games

- Any problem of Game without a Saddle Point is considered to be the problem of Mixed Strategy. In this cases both players will use different strategies with certain probabilities to optimize.
- here we need to find out the probabilities of various strategies of both the players as well as expected value of the game.

Games with Mixed Strategy are solved by the following methods depending on the size of the Payoff Matrix.

- (2 × 2) Game – Odds Method or Arithmetic Method
- Dominance Method (applicable for m × n Payoff Matrix convertible to 2 × 2 Payoff Matrix by applicationof Rules of Dominance)
- (2 × n) and (m × 2) Game – Graphical Method

1. Odds Method

- Odds Method is applicable if and only if the Payoff Matrix is of size (2 × 2).
- Odds are nothing but the magnitude(i.e. without sign or ignoring negative sign, if any) of the differences of the elements of various rows as well as columns.

Method of calculating Odds is given below –

1. Find out magnitude of difference in the values of cell (1,1) and (1,2) of the 1st Row and place it against the2nd Row.
2. Compute magnitude of the difference in the cell entries of (2,1) and (2,2) of the 2nd Row and put it againstthe 1st Row.
3. Compute magnitude of the difference in the cell entries of (1,1) and (2,1) of the 1st Column and put it belowthe 2nd Column.
4. Compute magnitude of the difference in the cell entries of (1,2) and (2,2) of the 2nd Column and put it below the 1st Column.
5. Ensure that the sum of the differences calculated for the Rows is equal to that for the columns. In other wordsSum of the differences calculated in steps (1) and (2) should be equal to that calculated in steps (3) and (4).

Note – Only the magnitude of the differences should be taken into account ignoring the negative signs, if any.

		Strategies of Y		
		Y_1	Y_2	ODDs
Strategies of X	X_1	a_1	a_2	$(b_1 - b_2)$
	X_2	b_1	b_2	$(a_1 - a_2)$
	ODDs	$(a_2 - b_2)$	$(a_1 - b_1)$	

Probabilities of X as well as Y taking different strategies are calculated by using the following formulae –

$P(X_1) = (b_1 - b_2) \div [(b_1 - b_2) + (a_1 - a_2)]$ and

$P(X_2) = (a_1 - a_2) \div [(b_1 - b_2) + (a_1 - a_2)]$

$P(Y_1) = (a_2 - b_2) \div [(a_2 - b_2) + (a_1 - b_1)]$ and

$P(Y_2) = (a_1 - b_1) \div [(a_2 - b_2) + (a_1 - b_1)]$

Value of the Game is determined using the formula:-

$v = [a_1(b_1 - b_2) + b_1(a_1 - a_2)] \div [(b_1 - b_2) + (a_1 - a_2)]$

[Note: $P(X_1) + P(X_2) = 1$ and $P(Y_1) + P(Y_2) = 1$. So once $P(X_1)$ is calculated, $P(X_2)$ can always be calculated as complement of $P(X_1)$ instead of going for the formula. Similar is the case for $P(Y_2)$.]

2. Dominance Method

Dominance Method is applied for reducing the size of (m × n) Payoff Matrix (when either one of m and n or both m and n are greater than 2) when there exist no Saddle Point. The aim is to get (2 × 2) Matrix, so that Odds Method can be applied to find the Probabilities and the Value of the Game.

It can be mentioned that the strategies which are dominated by the others and ultimately ignored will not be used by the players and hence their probabilities will be zero.

Principle of Dominance

- According to the Principle of Dominance if any strategy of a player dominates over his another strategy in all conditions then the later can be ignored.
- From the gainer's point of view, if a strategy gives more gain than another strategy for all strategies of the loser, then the first strategy dominates over the other and the second one can be ignored altogether.
- From the loser's point of view, if a strategy involves lesser loss than the other in all conditions, the second one can be omitted without affecting decision. So determination of superior or inferior strategy depends upon the objective of the player.
- Since each player has to select his best strategy, the inferior strategies can be eliminated..

For deleting the ineffective rows and columns, the following Rules are used –

Rule 1 – If all the elements of a row (say ith row) of a payoff matrix are less than or equal to the corresponding elements of another row (say jth row) then the Maximising Player will never choose the ith strategy. In other words ith strategy is dominated by the jth strategy.

Rule 2 – If all the elements of a column (say pth column) of a payoff matrix are more than or equal to the corresponding elements of another column (say qth column) then the Minimising Player will never choose the pth strategy. In other words, pth strategy is dominated by the qth strategy.

Rule 3 – A pure strategy may be dominated if it is inferior to average of two or more other pure strategies.

If all the elements of a row are less than or equal to the average of the corresponding elements of two or more other rows then this row is said to be dominated by the other group of rows for which average is computed.

Similar concept is also applicable for column with the exception of having its elements more than

the average of the corresponding elements of two or more column

- Principle of Dominance can be applied to both Pure Strategy as well as Mixed Strategy problems. Its basic objective is to reduce the size of the given Payoff Matrix.
- Aim should always be made to get a (2 × 2) matrix by using this Principle.

3. Graphical Method

Graphical Method is applied to solve (2 × n) and (m × 2) Game problems, when both m and n are more than 2. Since the optimal strategies for both the players assign non zero probabilities to the same number of pure strategies, it is obvious that if one player has only two strategies the other will also use two strategies.

Graphical method facilitates to find out which of the two strategies can be used.

When Rules of Dominance cannot be applied to a payoff matrix of size (2 × n) or (m × 2) then Graphical Method is used.

Following are the steps for solving a (2 × n) Game –

1. Draw two vertical lines 1 unit apart along a horizontal line to represent the axes $x_1 = 0$ and $x_1 = 1$ & mark a suitable scale on each one.
2. Take the values in the first Row of the Payoff Matrix and plot each one as a point on the scale of the vertical line $x_1 = 1$.
3. Take the values in the second Row of the Payoff Matrix & plot each one as a point on the scale of the vertical line $x_1 = 0$.
4. The point a_{1j} on the line $x_1 = 1$ should be joined to the point a_{2j} on the line $x_1 = 0$ to get a straight line.
5. Draw n such straight lines for j = 1, 2, 3,.....n. Each of these lines represents the expected payoff of the maximising player (whose 2 strategies are represented by the rows) against n different strategies of the minimising player (whose strategies are represented by the columns).
6. Mark the lower envelope of the area obtained by drawing these n straight lines.
7. The highest point of the lower envelope is the Maximin point.
8. The straight lines passing through this Maximin point corresponds to the optimum strategies of the minimising player. All the other strategies of the minimising player should be ignored.
9. So now the desired (2 × 2) payoff matrix is obtained, the Game can be solved using the method of Odds.

SMQs

Illustration-1

Solve the Game with the Payoff Matrix $\begin{bmatrix} 1 & 5 \\ 4 & 2 \end{bmatrix}$

Illustration-2

The Management of a company is negotiating with its Union for revision of hourly wages of its employees. The Management deployed a Consulting Firm who has prepared a payoff matrix for the purpose which indicates the additional hourly cost (in `) to the company. It is shown below: you being a part of the Consulting Firm have to assist the Management in selecting the best strategy. What is the value of the game? How is it going to affect the company's cost?

Management's Strategies	Strategies of the Union			
	U_1	U_2	U_3	U_4
M_1	2.50	2.70	3.50	- 0.20
M_2	2.00	1.60	0.80	0.80
M_3	1.40	1.20	1.50	1.30
M_4	3.00	1.40	1.90	0

Illustration-3

Solve the Game using Dominance Principle $\begin{bmatrix} 15 & 2 & 3 \\ 6 & 5 & 7 \\ -7 & 4 & 0 \end{bmatrix}$

Illustration-4

Joy Givers and Milan Toys are the two toy manufacturers who always compete with each other to increasetheir respective market shares. For both the companies the Marketing team work with close coordination with theDesign team and always come out with attractive toys which are normally in great demand. To meet the demand, they have various strategic options like working for 8 hours a day, 12 hours a day, 16 hours a day, 24 hours a day,Subcontracting etc. which will ultimately increase the market share. Joy Givers have decided not to go for all the above mentioned options and set up the following payoff matrix in which the percentage increase in market share is given against different strategies of Milan Toys

STRATEGIES of Joy Givers	Milan Toys			
	Working 8 hrs/day	Working 12 hrs/day	Working 16 hrs/day	Subcontracting
Working 12 hrs/day	8	10	9	14
Working 16 hrs/day	10	11	8	12
Working 24 hrs/day	13	12	14	13

Use Principle of Dominance to find the Optimal Strategies of the two manufacturers and the value of the Game.

Illustration-5

Solve the Game represented by the payoff matrix : $\begin{bmatrix} 1 & 3 & 12 \\ 8 & 6 & 2 \end{bmatrix}$

- **Multiple Choice Questions**

1. Two person zero sum game means that
   (a) The sum of losses of one player is equal to the sum of the gains of the other
   (b) The sum of losses of one player may not be equal to the sum of the gains of the other

(c) No player gains or loses

(d) None of the above

2. Game theory models are classified by the

(a) Number of players (c) Number of strategies

(b) Sum of all payoffs (d) All of these

3. A game is said to be unfair if

(a) Upper and lower values of the game are not equal

(b) Upper and lower values of the game are equal and the sum is zero

(c) Option (a) is correct but not Option (b)

(d) Option (b) is correct but not Option (a)

4. What happens when the maximin and minimax values of the game are equal?

(a) No solution exists (c) Saddle point exists

(b) Solution is mixed (d) None of these

5. A mixed strategy game can be solved by

(a) Arithmetic method (c) Dominance method

(b) Graphical method (d) All of these

6. The size of the payoff matrix of a game can be reduced by using the principle of

(a) Game inversion (c) Dominance

(b) Rotation reduction (d) Game transpose

7. The payoff value for which each player in a game always selects the same strategy is called the

(a) Saddle point (c) Both option (a) and option (b)

(b) Equilibrium point (d) None of the above

8. Games which involve more than two players are called

(a) Conflicting games (c) N person game

(b) Negotiable games (d) All of these

9. When the sum of the gains of one player is equal to the sum of the losses to another player then it is called

(a) Fair game (c) Both option (a) and option (b)

(b) Zero sum game (d) Only option (b) and not option (a)

10. When no saddle point is found in the payoff matrix of a game, the value of the game is found by

(a) Reducing the size of the game to apply the odds method

(b) Solving any one of the (2×2) sub game

(c) Finding the average of the values of the payoff matrix

(d) None of these

11. A saddle point exists when

(a) Maximin value = Maximax value
(c) Minimax value = Maximin value
(b) Minimax value = Minimum value
(d) Minimax value = Minimin value

12. In a pure strategy game

(a) Any strategy can be selected arbitrarily
(b) A particular strategy is selected by each player
(c) Both players select their optimal strategy
(d) None of these

13. In a mixed strategy game

(a) saddle point exists
(b) Each player selects the same strategy without considering the choice of the other
(c) Each player always selects the same strategy
(d) None of these

14. Game theory is the study of

(a) Selecting optimal strategies
(b) Resolving conflict between players
(c) Giving equal outcome to the participants
(d) None of the above

15. If the value of the game is zero, then the game is known as

(a) Fair strategy
(c) Pure game
(b) Pure strategy
(d) Mixed strategy

16. The games with saddle points are

(a) Probabilistic in nature
(c) Stochastic in nature
(b) Normative in nature
(d) Deterministic in nature

17. When the game is played on a predetermined course of action, which does not change throughout the game then it is known as

(a) Pure strategy game
(c) Mixed strategy game
(b) Fair strategy game
(d) Unsteady game

18. If the losses of Player A are the gains of Player B, then it is called

(a) Lump sum game
(c) Unfair game
(b) Zero sum game
(d) None of the above

19. Identify the incorrect one

(a) A game without saddle point is probabilistic
(b) Game with saddle point will have pure strategies

(c) Game with saddle point cannot be solved with dominance rule

(d) Game without saddle point has mixed strategies

20. In case there is no saddle point in a game then the game is

(a) Deterministic game (c) Mixed strategy game

(b) Fair game (d) Multi player game

21. When Minimax and Maximin criteria matches then

(a) A fair game exists (c) Mixed Strategy exists

(b) An unfair game exists (d) Saddle point exists

22. When there is dominance in a game then

(a) Least of the row ≥ Highest of another row

(b) Least of the row ≤ Highest of another row

(c) Every element in a row ≥ Corresponding element of another column

(d) Every element in a row ≤ Corresponding element of another row

23. A game is played when

(a) The manager gives signal

(b) Each player chooses one of his courses of action simultaneously

(c) The player who comes to the field first says he will start the game

(d) When the latecomer starts the game

24. In a game the list of the courses of action with each player is

(a) Finite (c) Only 3

(b) Infinite (d) None of the above

25. When the game is having a saddle point then the method used to solve the game is

(a) Linear Programming method (c) Odds method

(b) Minimax and Maximin criteria (d) Graphical method

26. Linear Programming method should be used to determine the value of the game when the size of the payoff matrix is

(a) 2×2 (c) m×2

(b) 3×4 (d) 2×n

27. If there are more than two persons in a game then the game is known as

(a) Non zero sum game (c) Multiplayer game

(b) Open game (d) Big game

28. A competitive situation is known as

(a) Competition (c) Game

(b) Marketing (d) All the above

29. Which one of the following is an assumption of Game Theory?

(a) All players act rationally and intelligently
(b) The winner alone acts rationally
(c) The loser acts intelligently
(d) Both believes in luck

30. For the Payoff Matrix $\begin{bmatrix} -5 & -2 \\ 10 & 5 \end{bmatrix}$ the maximising player always uses

(a) The first strategy
(b) Average of the two strategies
(c) The second strategy
(d) All the above strategies

Answers:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
a	d	a	c	d	c	a	c	d	a	c	c	a	a	c
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
d	a	b	c	c	d	d	b	a	b	b	c	c	a	c

Comprehensive Numerical Problems

1. Find the optimal strategies of the Players for the game having payoff matrix. What is the value of the Game?

Strategies	B_1	B_2	B_3	B_4
A_1	1	7	3	4
A_2	5	6	4	5
A_3	7	2	0	3

2. Solve the game $\begin{pmatrix} 5 & 1 \\ 3 & 4 \end{pmatrix}$

3. Reduce the following game by Dominance Rules and Solve it.

		Strategies of Minimising Player				
		P	Q	R	S	T
Strategies of Maximising Player	I	1	3	2	7	4
	II	3	4	1	5	6
	III	6	5	7	6	5
	IV	2	0	6	3	1

4. Solve the game $\begin{matrix} 3 \\ 3 \\ 2 \\ -2 \end{matrix}\begin{matrix} \\ -4 \\ 5 \\ 8 \end{matrix}$ using Graphical method

5. In the suburban area of a large city there are two stores Laxmi Bhandar and Goswami Stores who handle sundry goods. The total number of customers is eqally divided between the two due to the fact that the price, quality of goods, services etc. of the two are at par. Assume that a gain of customers for Laxmi Bhander is a loss to Goswami Stores and vice versa. Both the stores plan to run annual sales during the festival period of the year. Sales are advertised through Social Media, Cable TV local channel and Printed Leaflets. Based

on the past experience, Laxmi Bhander has prepared the following payoff matrix for the gain or loss in percentage of customers for its different strategies against various counter strategies of Goswami Stores.

	Strategies of Goswami Stores		
Strategies of Laxmi Bhander	Printed leaflets	Cable TV	Social media
Printed leaflets	30	40	- 80
Cable TV	0	15	- 20
Social media	90	20	50

Determine the optimal strategies and worth of such strategies for the stores. What is meant by the cell entry – 80 in the above payoff matrix?

6. Two competing firms (A and B) produce consumer goods of different kind. Among the products one is considered as their bread and butter in terms of the revenue generated. Both the firms are very cautious about the market share for this particular product and keep on doing advertisement campaigns throughout the year to retain the existing customers and also to attract the new ones. For this the marketing teams of both work round the clock and that of A developed data corresponding to varying degrees of advertisement. Same is given below:

(a) If both the firms take same strategy to counter each other then their market share will be equal.

(b) Against firm A's strategy of "No marketing" if B goes for "Medium marketing" then A's share of the market will be 40%. For the same strategy of A the market share will be 28% if B takes the strategy "Large marketing"
Against firm A's strategy of "Medium marketing" if B goes for "No marketing" then A's share of the market will be 70%. For the same strategy of A the market share will be 45% if B takes the strategy "Large marketing

(a) Against firm A's strategy of "Large marketing" if B goes for "No marketing" then A's share of the market will be 75%. For the same strategy of A the market share will be 47.5% if B takes the strategy "Medium marketing"

Based on the above information prepare the Payoff Matrix. Solve the game problem to get the optimal strategies of the player A. What is the value of the game?

7. Using the data of the above problem prepare the Payoff Matrix for A when you are supplied with the following information.

(c) Selling price of the product = ` 4 per unit
(d) Variable cost of the product = ` 2.50 per unit
(e) Annual cost for Medium advertising = ` 5000
(f) Annual cost for Large advertising = ` 15000
(g) Annual sales volume of the product for Firm A = 30000 units What advertising policy should firm A pursue?

Hints-

Find out the Annual sales volume, for different combination of strategies of A and B. As an example, Annual Sales volume corresponding to A's strategy of "Medium advertisement" and B's strategy of "Large

advertisement" is 45% of 30000 = 13500 units
Calculate Annual Profit to the Firm A using the formula below for various combination of strategies of A and B.
Annual Profit = (Selling price – Variable cost) × Annual Sales volume – Annual cost of advertisingExample of this calculation is:-
For A's strategy of "Medium advertisement" and B's strategy of "Large advertisement" the Annual Profit of Firm A is (4 – 2.5) × 13500 – 5000 = ` 15250/-
When the Profit figures for all the combinations of strategies of A and B are calculated then the followingpayoff matrix is obtained.

	Strategies of B		
Strategies of A	No advertising	Medium advertising	Large advertising
No advertising	22500	18000	12600
Medium advertising	26500	17500	15250
Large advertising	18750	6375	7500

From the above matrix we find, against the various strategies of A, the minimum profit figures are as follows – For No advertising – ` 12600
For Medium advertising – ` 15250For Large advertising – ` 6375
Thus, to maximise the minimum profit, A should opt for Medium advertising and spend ` 5000 per annum.

Answers:

1. Optimal strategies A_2 and B_3. Value of the game = 4
2. Strategies of the Maximising Player = (1/5, 4/5) & for the Minimising Player = (3/5, 2/5). Value of the game = 17/5
3. Optimal strategy for the Maximising Player is III and that for the Minimising Player is Q. Value of the game = 5
4. Optimal strategy of the Maximising Player (0.3, 0.7, 0) and for the Minimising Player (0.9, 0.1). Value = 2.3
5. Optimal strategies of Laxmi Bhander = (1/5, 0, 4/5) and for Goswami Stores = (0, 13/15, 2/15), Value of the game = 24 Cell entry (– 80) means when Laxmi Bhander will take the strategy of distributing Printed Leaflets against the counter strategy of Goswami Stores of Social Media advertisement then they will lose 80% of their customer which will be gained by Goswami Stores.
6. The payoff matrix Showing A's market share is –

	Strategies of B		
Strategies of A	No advertising	Medium advertising	Large advertising
No advertising	50	40	28
Medium advertising	70	50	45
Large advertising	75	47.5	50

Probabilities of A's strategies are (0, 1/3, 2/3). Value of the game = 145/3 = 48.3 Thus, A can expect to have 48.3% market share.

Simulation

Introduction

1) There are various models that supports decision making.
2) Most of the models has underlying assumptions and the model will work only when the underlying assumptions are satisfied in real world situation.
3) Where no model can be applied in a decision making situation, then we should make decision based on chance. The scientific way of making decision based on chance is called 'Simulation'.
4) Simulation is a process through which we build a model and test it through random numbers to understand how it works.
5) Random numbers represent the chance factor and is closely related to probabilities of events.

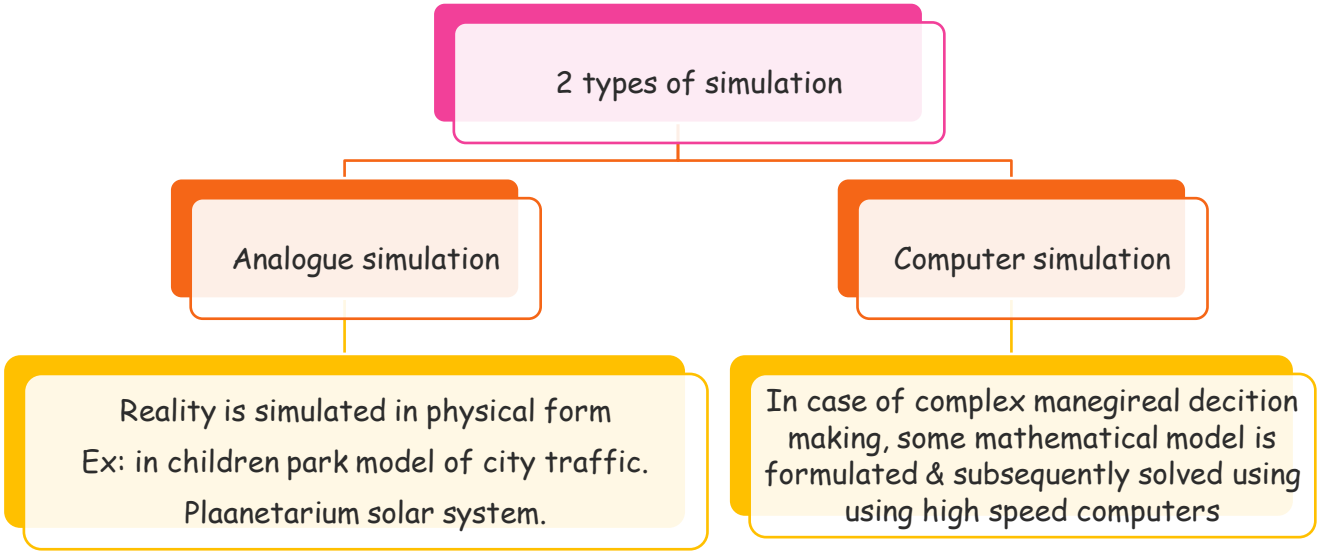
STEPS IN SIMULATION PROBLEM:

In any simulation problem there are two steps:

(i) Random Number Coding → Each event is coded with a range of Random Numbers based onits probability.
(ii) Simulation Work Sheet (or) Fitting Random Numbers → The Random Numbers selected isfitted to the code to identify the decision.

The above process is popularly called "Monti Carlo Simulation:

Types of simulation



Simulation models:

1. Deterministic model	In this case input and out put variables are not random instead they described by exact functional relationship
2. Probabilistic model	Here method of random sampling is used (monte Carlo technique)
3. Static model	These models don't take variable time into consideration
4. Dynamic model	These models deal with time varying interaction

Illustration-2

The past data of demand per week (in '00 kgs.) of a confectionery item is given below –

Demand/Week	**0**	**5**	**10**	**15**	**20**	**25**
Frequency	**2**	**11**	**8**	**21**	**5**	**3**

Using the sequence of random numbers – 35, 52, 13, 90, 23, 73, 34, 57, 35, 83, 94, 56, 67, 66 generate thedemand for the next 10 weeks. Also find out the average demand per week.

KEY NOTES:

Illustration-3

The manager of a book store has to decide the number of copies of a particular tax law book to order. A book costs Rs 60 and is sold for Rs 80. Since some of the tax laws change year after year, any copies unsold while the edition is current must be sold for Rs 30. From past records, the distribution of demand for this book has been obtained as follows:

Demand (No of copies)	15	16	17	18	19	20	21	22
Proportion	0.05	0.08	0.20	0.45	0.10	0.07	0.03	0.02

Using the following sequence of random numbers, generate the demand for 20 time periods (years). Calculate the average profit obtainable under each of the courses of action open to the manager. What is the optimal policy?

14	**02**	**93**	**99**	**18**	**71**	**37**	**30**	**12**	**10**
88	**13**	**00**	**57**	**69**	**32**	**18**	**08**	**92**	**73**

KEY NOTES:

Illustration-4

A Small retailer has studied the weekly receipts and payments over the past 200 weeks and has developed the following set of information

Weekly Receipts (Rs)	Probability	Weekly Payments (Rs)	Probability
3000	0.20	4000	0.30
5000	0.30	6000	0.40
7000	0.40	8000	0.20
12000	0.10	10000	0.10

Using the following set of random numbers, simulate the weekly pattern of receipts and payments for the 12 weeks of the next quarter, assuming further that the beginning bank balance is Rs. 8000. What is the estimated balance at the end of the 12 week period? What is the highest weekly balance during the quarter? What is the average weekly balance for the quarter?

Random Numbers

For Receipts	**03**	**91**	**38**	**55**	**17**	**46**	**32**	**43**	**69**	**72**	**24**	**22**
For Payments	**61**	**96**	**30**	**32**	**03**	**88**	**48**	**28**	**88**	**18**	**71**	**99**

KEY NOTES:

Illustration-5

Patients arriving at a village dispensary are treated by a doctor on a first-come-first-served basis. The inter-arrival time of the patients is known to be uniformly distributed between 0 and 80 minutes, while their service time is known to be uniformly distributed between 15 and 40 minutes. It is desired to simulate the

system and determine theaverage time a patient has to be in the queue for getting service and the proportion of time the doctor would be idle.

Carry out the simulation using the following sequences of random numbers. The numbers have been selected between 00 and 80 to estimate inter-arrival times and between 15 and 40 to estimate the service times requiredby the patients.

Series 1	**07**	**21**	**12**	**80**	**08**	**03**	**32**	**65**	**43**	**74**
Series 2	**23**	**37**	**16**	**28**	**30**	**18**	**25**	**34**	**19**	**21**

KEY NOTES:

Illustration-6

A businessman is considering taking over a certain new business. Based on past information and his own knowledge of the business, he works out the probability distribution of the monthly costs and sales revenues, as given here:

Cost (in Rs)	Probability	Sales Revenue (Rs)	Probability
17000	0.10	19000	0.10
18000	0.10	20000	0.10
19000	0.40	21000	0.20
20000	0.20	22000	0.40
21000	0.20	23000	0.15
		24000	0.05

Use the following sequences of random numbers for estimating costs and revenues. Obtain the probability distribution of the monthly net revenue.

Sequence 1	82	84	28	82	36	92	73	91	63	29
(for Cost)	27	26	92	63	83	02	10	39	10	10
Sequence 2	39	72	38	29	71	83	19	72	92	59
(for Revenue)	49	39	72	94	04	92	72	18	09	00

KEY NOTES:

Illustration 32.

After observing heavy congestion of customers over a period of time in a petrol station, Mr. Petro has decided to set up a petrol pump facility on his own in a nearby site. He has compiled statistics relating to the potential customer arrival pattern an service pattern as given below. He has also decided to evaluate the operations by using the simulation technique.

Arrivals		Services	
Inter-arrival time (minutes)	Probability	Inter-arrival time (minutes)	Probability
2	0.22	4	0.28
4	0.30	6	0.40
6	0.24	8	0.22
8	0.14	10	0.10
10	0.10		

Assume:

i) The clock starts at 8:00 hours

ii) Only one pump is set up.

iii) The following12 Random Numbers are to be used to depict the customer arrival pattern:
78, 26, 94, 08, 46, 63, 18, 35, 59, 12, 97 and 82.

iv) The following 12 Random Numbers are to be used to depict the service pattern:
44, 21, 73, 96, 63, 35, 57, 31, 84, 24, 05, 37

You are required to find out the

i) probability of the pump being idle, and

ii) Average time spent by a customer waiting in queue.

Solution:

	Inter-arrival time				Service time		
Minutes	Probability	Cumulative probability	Range	Minutes	Probability	Cumulative probability	Range
2	.22	.22	00-21	4	.28	.28	00-27
4	.30	.52	22-51	6	.40	.68	28-67
6	.24	.76	52-75	8	.22	.90	68-89
8	.14	.90	76-89	10	.10	1.00	90-99
10	.10	1.00	90-99				

Sl. No.	Random No. for inter arrival	Inter arrival time	Entry time in queue	Service start time	Random no for service.	Service time	Service end time	Waiting time of customer	Idle time
1	78	8	8.08	8.08	44	6	8.14	-	8
2	26	4	8.12	8.14	21	4	8.18	2	-
3	94	10	8.22	8.22	73	8	8.30	-	4
4	08	2	8.24	8.30	96	10	8.40	6	-
5	46	4	8.28	8.40	63	6	8.46	12	-
6	63	6	8.34	8.46	35	6	8.52	12	-
7	18	2	8.36	8.52	57	6	8.58	16	-
8	35	4	8.40	8.58	31	6	9.04	18	-
9	59	6	8.46	9.04	84	8	9.12	18	-
10	12	2	8.48	9.12	24	4	9.16	24	-
11	97	10	8.58	9.16	05	4	9.20	18	-
12	82	8	9.06	9.20	37	6	9.26	14	-
Total Time								140	12

Average waiting time spent by the customer = 140 / 12 = 11.67 minutes

Probability of idle time of petrol station = 12/86 = 0.1395

Multiple Choice Questions

1. Which of the following is first step for performing Simulation analysis?
   (a) Choose input variables.
   (b) Create entities for the simulation process.
   (c) Prepare a problem statement.
   (d) Determine the output variables.
2. Which of the following are the advantages of using Modelling and Simulation?
   (a) Easy to understand. (c) Easy to upgrade.
   (b) Easy to test. (d) All of the above.
3. Which one of the following is not an application area of Modelling and Simulation?

(a) Military applications
(b) Designing semiconductors
(c) Telecommunications
(d) Food industry

4. Which of the following is the first step for developing the Simulation Model?
   (a) Design the problem
   (b) Identify the problem.
   (c) Collect and start processing the system data
   (d) Develop the model using Network diagram.

5. Simulation is the process of using a model to study the performance of a system.
   (a) Agreeable
   (b) Not agreeable.
   (c) Partly agreeable
   (d) Cannot comment.
6. Disadvantage of using Modelling and Simulation lies in the statement –
   (a) Simulation requires manpower and it is a time consuming process.
   (b) Simulation results are difficult to translate and only experts can understand it.
   (c) Simulation is an expensive process.
   (d) All of the above.
7. Monte Carlo Simulation gets its name from which of the following?
   (a) Data collection.
   (b) Model formulation
   (c) Random number assignment
   (d) Analysis
8. Select the valid reasons for using Simulation.
   (a) Relationship between the variables is non-linear.
   (b) Optimized solutions are obtained.
   (c) Conduct experiment without disrupting the real system.
   (d) Both (a) and (c)
9. Simulation uses logical relationship and mathematical expressions of the –
   (a) Real system
   (b) Computer model
   (c) Performance measures
   (d) Inferences
10. While assigning random numbers in Monte Carlo Simulation, it is –
    (a) Not necessary to assign the exact range of random number interval as the probability.
    (b) Necessary to develop a cumulative probability distribution.
    (c) Necessary to assign the particular appropriate random numbers.
    (d) All of the above.

11. Random numbers are used –
    (a) To give random outcomes.
    (b) To describe the uncertainty of the input values.
    (c) To assign values to the parameters.
    (d) To change the problem solution.
12. Simulation can keep track of several different kinds of information, including orders, inventory,

financial planning

(a) Correct
(b) Partly correct
(c) Totally incorrect
(d) None of the above

13. Monte Carlo Simulation ensures that –
    (a) The simulated probability distribution will be the same as the actual probability distribution.
    (b) Only one uncertain decision can be taken in any simulation model.
    (c) Probabilities will have at most two decimal place values.
    (d) Each one of the above statement is true.
14. What should a Project Manager do when his preliminary task estimate differs from Monte Carlo Simulation?
    (a) Perform more Monte Carlo Simulation runs.
    (b) Use this information to assess project uncertainty and risk and to review and possibly change crucial variables.
    (c) Scrap the project as untenable.
    (d) Discuss these differences at future project team meetings.
15. What can be expected at the end of a Monte Carlo Simulation exercise?
    (a) A conservative outcome.
    (b) A decision backed by number that makes the decision maker confident about moving forward.
    (c) All outcomes from all possible probabilities exercise.
    (d) None of the above.
16. Analytical results are taken into consideration before a simulation study so as to -
    (a) Identify suitable values of the system parameter
    (b) Determine the optimal decision.
    (c) Identify the suitable values of the decision variables for the specific choice of the system.
    (d) All of the above.

17. Which of the following statements are applicable for Pseudo Random Numbers?
    (a) They are numbers similar to Random Numbers.
    (b) They are generated by using Remainder Method.
    (c) They satisfy the statistical test for randomness.
    (d) All of the above.
18. Large complicated simulation models are appreciated because –
    (a) Their average costs are not well defined
    (b) It is difficult to create the appropriate events
    (c) Both (a) and (b) are true.
    (d) None of the above is true.
19. Simulation should not be applied in all cases because it –
    (a) Requires considerable talent for model building and extensive computer programming efforts.
    (b) Consumes too much computer time.
    (c) Provides at best an approximate solution to the problem.
    (d) All of the above.

20. Simulation is defined as –
    (a) A technique that uses computers.
    (b) An approach for reproducing the processes by which events by chance and changes are created by computers.
    (c) A procedure for testing and experimenting on models to answer what if, then so and so type questions.
    (d) All of the above.
21. Few causes of simulation analysis failure are –
    (a) Inadequate level of user participation.
    (b) Inappropriate levels of detail.
    (c) Incomplete mix of essential skills.
    (d) All of the above.
22. To make simulation more popular –
    (a) Large cost overruns need to be avoided.
    (b) Prolonged delays need to be avoided.
    (c) User dissatisfaction with simulation results need to be avoided.
    (d) All of the above.
23. The important step required for simulation approach in solving a problem is –
    (a) Test and validate the model.
    (b) Design the experiment.
    (c) Conduct the experiment.
    (d) All of the above.
24. The general purpose system simulation language –
    (a) Requires program writing
    (b) Does not require program writing.
    (c) Requires predefined coding forms.
    (d) Needs a set of equations to describe a system.

25. An advantage of simulation as opposed to optimization is that –
    (a) Several options of measure of performance can be examined.
    (b) Complex real life problems can be studied.
    (c) It is applicable to cases where there is randomness in a system
    (d) All of the above.
26. The purpose of using simulation technique is to –
    (a) Imitate a real world situation.
    (b) Understand properties and operating characteristics of complex real life problems.
    (c) Both (a) and (b) above.
    (d) Only (b) but not (a).
27. As simulation is not an analytical model, its solution should be viewed as –
    (a) Exact
    (b) Unrealistic
    (c) Approximation.
    (d) Simplified

28. All of the following are advantages of simulation except –
    (a) Facilitates in finding the optimal solution.
    (b) It is a low cost process.
    (c) It deals with playing Games.
    (d) Time compression is an issue with it.

29. The drive up window of a fast food operation was being studied using simulation for a variety of operating characteristics. As part of the study data was collected on Order Processing Time as given in the following table. Using the first two digits of the Random Numbers, determine the processing time that would be used to simulate the fifth sample.

Processing time (Minutes)	**1**	**2**	**3**	**4**
Relative Frequency	**0.30**	**0.45**	**0.20**	**0.05**

Customer	1	2	3	4	5	6	7	8	9	10
Random No.	1048	2236	2413	4216	3757	1501	4657	4836	9309	3997

    (a) 2 minutes
    (b) 4 minutes
    (c) 1 minute
    (d) 3 minutes

30. The drive up window of a fast food centre was being studied using simulation for a variety of operating characteristics. As part of the study data was collected on Customer Arrivals as given in the following table. Using expected value calculations determine the expected time between customer arrivals.

Inter arrival time (Minutes)	0.5	1.0	2.0	3.0	4.0	5.0	6.0
Probability	0.10	0.25	0.20	0.30	0.05	0.05	0.05

    (a) 2.35 minutes
    (b) 2.00 minutes
    (c) 2.70 minutes
    (d) 1.65 minutes

Answers:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
c	d	d	b	a	d	c	d	a	b	b	a	a	b	b
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
c	d	b	d	d	d	d	d	b	d	c	c	b	a	a

Network Analysis-PERT, CPM

Learning Objectives

1) Understanding some basic terms used in Network
2) Learning to draw a network diagram and appropriately use dummy activates
3) Forward Pass and Backward Pass Procedure
4) Calculating EST, LST, EFT and LFT of each activity
5) Total float, free float and independent float
6) Identification of critical activates and critical path
7) Program evaluation review technique (PERT)
8) Network Crashing

Introduction

A. Projects involves huge investments, complex activities and a longer time frame.
B. To avoid time & cost over-run it is necessary to properly plan & control the project implementation.
C. One of the popular techniques used in project planning and control is "Network Analysis".
D. In Network Analysis we,
   a) Break the projects into number of activities
   b) Sequence the activities
   c) Present the same in the form of a network diagram which is a pictorial representation of theentire project.
   d) Estimate the time and resource required for each activity and fit it into the network diagram.
   e) Then use techniques like crashing, resource allocation etc., to manage the network.

Understanding some basic terms used in Network

1) Activities
2) Events
3) Types of Activities
4) Types of Events
5) Errors in drawing a Network
6) Conventions in Network Diagram

Activities & Events

1)

a) Activity is something that consumes time and resource.

b) It is represented as a straight line in the Network.

Here Activity 'A' is covered by 2 circles ① and ②. They are called "Events.

c) Event ① is the starting event of Activity 'A' and called "Tail Event" and event ② is the ending event of an activity and called "Head Event".

2) Types of Activities:

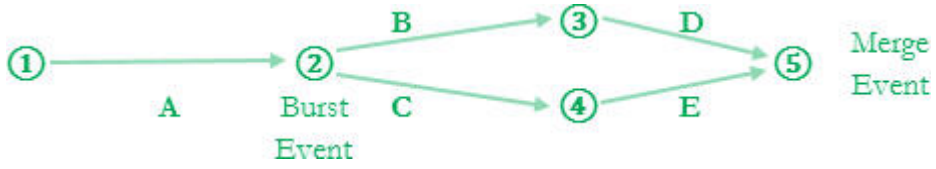
The diagram shows Activity 'A' should be done first and when Activity 'A' is over Activity 'B' and Activity 'C' can start. Here there are 3 types of activities:

a) Preceding Activities → For Activity 'B' and Activity 'C', Activity 'A' is proceeding.

b) Succeeding Activities → For Activity 'A', Activity 'B' and Activity 'C' are succeeding.

c) Simultaneous Activities → For Activity 'B' Activity 'C' is simultaneous and vice-versa.

3) Events:



a) When an event is a starting event for more than one activity it is called "Burst Event". Here Event ② is starting event for Activity 'B' and Activity 'C'.

b) If an event is ending event of more than one activity it is called "Merge Event". Event ⑤ represents completion of Activity 'D' and Activity 'E'.

Errors in Networking

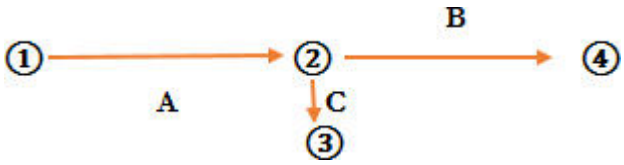
1) Looping Error
2) Dangling Error
3) Mistake in Succeeding, Preceding relationship

Looping Error

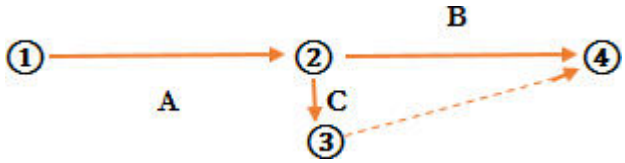
1) When Activity 'A' is over Activity 'B' will starts, when Activity 'B' is over Activity 'C' starts, oncompletion of Activity 'C' again Activity 'B' starts. Thus a loop is formed.
2) This project will never end.

In real world there are situations where finite loops can be formed which is supported by flowchartingand programming techniques. However, in network looping is not possible because with loops we cannot perform the Forward and Backward Pass procedures.

Dangling Error



1) A project can have only one starting event and one ending event.
2) In the above diagram if we call event ③ as the completing event of the project then Activity 'B' becomes irrelevant for the project completion and vice-versa.
3) This error is called Dangling error. Every activity should either be connected to next Activity or to thelast event.
4) The error can be removed by using 'Dummy Activity'.
5) Dummy activity is an activity that consumes '0' time and '0' resource. It is represented by dotted lines.

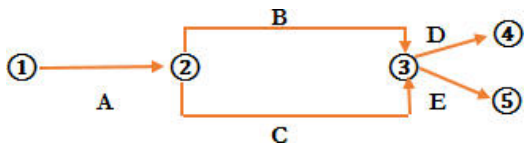


6) Event ④ has now become merge event. It represents the completion of activities 'B' and dummy. Since dummy takes '0' time it gets completed as soon as Activity 'C' is completed. Hence event ④ represents completion of Activity 'B' and Activity 'C'.

Mistake in Preceding, Succeeding relationship

Example: A = 1st activity. B & C can start once A is completed. D can start once B & C is over. 'E' can start once B is over.

1) Wrong Diagram:

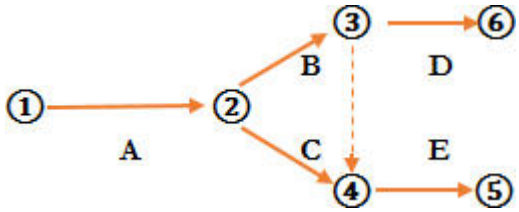


There are two mistakes in this diagram.

i) Two Activities can have a common tail event or can have common head event but cannothave the same head and tail events as it creates problems in Forward, Backward pass computations.

ii) For staring of Activity 'D', Activity 'B' and Activity 'C' should merge and for starting ofactivity 'E' activity 'B' alone should be completed.

The above can be resolved using dummy activity.



Conventions in Network Drawing

1) Time moves from left to right.
2) Activities are represented as straight lines.
3) Head event number should be greater than tail event number.
   Try to avoid crossing of lines while drawing network. If necessary, use gates.

Forward Pass Procedure:

1) Forward Pass is a procedure through which we find out earliest start time for every activity.
2) It is done as follows:
   a. Put "E" of event 1 as "0".
   b. "E" of head event = "E" of tail event + Duration
   c. Where the head event is merge event it will have multiple tails. In such case it is "E" of each tail event + Duration whichever is higher.

Backward Pass Procedure:

1) Through this procedure we find out latest finish time of each activity.
2) It is done as follows:

a. Assign "L" of the last event as "E" of last event.
b. "L" of tail event = "L" of Head event – Duration

If the tail event is burst event, it will have multiple heads. In such case it is "L" of each headevent – Duration whichever is lower.

Calculation of EST, EFT, LST, LFT and Total Float

Activity	t_n – Duration	Start		Finish		Total Float
		E	L	E	L	

Calculation of time estimates:

1) EST = "E" of Tail Event
2) LFT = "L" of Head Event
3) LST = "LFT – Duration
4) EFT = EST + Duration.

Total Float, Critical Activity and Critical Path:

1) Total Float = LFT – EFT (or) LST – EST
2) Total Float indicates by how many days we can postpone an activity without affecting the project duration.
3) Those activities having total float as '0' cannot be postponed and are referred as "Critical Activity".
4) The path in which all the activities are critical activates is called "Critical Path".
5) Critical path is the longest path in the Network and all the activities in the path are critical activates.

Paths	Duration
1 – 2 – 3 – 5 – 6	6 + 8 + 6 + 16 = 36
1 – 2 – 3 – 4 – 5 – 6	6 + 8 + 0 + 20 + 16 = 50
1 – 2 – 4 – 5 – 6	6 + 10 + 20 + 16 = 52

→ Critical Path

Step 6: Calculation of Free Float and Independent Float

Activity	Total Float	Free Float (Total Float – Head event slack)	Independent Float (Free Float – Tale event slack)

Notes:

1) Concept of free float:
   a) Free float = Total Float – Head Event Slack
   b) It is that part of total float which does not affect the float of succeeding activity.
2) Concept of independent float:
   a) Independent Float = Free Float – Tail Event Slack
   b) It is that part of that total float which does not affect the float of succeeding and precedingactivity.

Network Crashing – CPM (Critical path method)

1) Crashing is a process through which we try to reduce duration of a project.
2) The network crashing affects the project cost in two ways:
   (i) Benefit → Savings in Overhead Cost → Overhead costs are those costs which are linked to project duration
   (ii) Cost → To reduce the project duration we should reduce activity duration for which we should employ more resources in the activities. The cost involved is called "Crash Cost".
3) Keep Crashing the network as long as the benefit of crashing exceeds the cost of crashing. When this reverses stop crashing.
4) The project length (duration) at which the total cost is minimum is called "Optimum Project Length/Duration".
5) If the problem asks "Minimum Project Length/Shortest Project Duration", continue crashing till we reach a stage where further crashing not possible.

Normal duration – before crashing
Optimum duration- after crashing (after crashing – benefit > cost)
Shortest duration – cc > benefit

Notes:

1) When decision regarding crashing is made in the remarks column in evaluation table.
2) There are two aspects to be decide:
   (i) What activity to be crashed
   (ii) How many days to crash.

3) Always crash only critical activities because crashing non-critical activities does not result in reduction of project duration. It is a wasteful expenditure.
4) While selecting select least cost critical activity.
5) If there are more than one critical paths, all the paths should be simultaneously crashed. Here we have two options:
   (i) Crash an activity common to all the parts
   (ii) Crash one activity each from every critical path That option that gives lowest crash cost should be selected.
6) In deciding the number of days to be crashed we should consider 2 tables:
   (i) Slash Table → To see how many crash days are available
   (ii) Paths Table → To ensure that the path that being is crashed retains it's criticality.
   (iii) The rule is **"A path once critical should always be critical".**

   Continue crashing till we reach a stage where we don't have crash days available in any one of the critical path.

Program Evaluation Review Technique (PERT)

1) When the activity times for a project could not be estimated with certainty, Project Evaluation Review Technique should be used.
2) In Project Evaluation Review Technique 3 time estimates are made for every activity namely
   Optimistic time (+ve mind set) -T_o
   Pessimistic time (-ve mind set)- T_p
   Most likely time.-T_m
3) These estimates are assumed to fall into Normal Probability Distribution.
4) For Drawing the network which time should be taken? Is it optimistic, pessimistic or most likely?
   Answer: Neither of the 3. We should take **expected time** for the network.
5) Expected time is the average time where the weights for optimistic and pessimistic is '1' and most likely'4'.

   $t_e = \frac{t_o+4t_m+t_p}{6}$ where

   t_e = Expected time
   t_o = Optimistic time
   t_m = Most likely time
   t_p = Pessimistic time
6) When the time is expected (Mean) there should be a variance around mean. The variance of expected time $\sigma^2 = \left[\frac{T_p - T_o}{6}\right]^2$

$z = \frac{T_S - T_E}{\sigma}$,,,, T_S = stipulated time .

Illustration-1

The following table gives the activities and other relevant information related to "Making of a loaf".

Activity	Preceded by	Elapsed Time (Minutes)
A - Weigh ingredients	-	**1**
B - Mix ingredients	A	**3**
C - Dough rising time	B	**60**
D - Prepare tins	-	**1**
E - Pre-heat oven	-	**10**
F - Knock back dough and place in tins	C&D	**2**
G - 2nd dough rising time	F	**15**
H - Cooking time	**E & G**	**45**

Draw a Network diagram. Also find the Earliest and Latest Times of each Event of the Network. Identify the different paths of the Network and their corresponding durations. Which path is critical? Find the time required tocomplete the job.

KEY POINTS:

Illustration-2

Activity	Preceded by	Elapsed Time (Minutes)
A - Weigh ingredients	-	1
B - Mix ingredients	A	3
C - Dough rising time	B	60
D - Prepare tins	-	1
E - Pre-heat oven	-	10
F - Knock back dough and place in tins	C&D	2
G - 2nd dough rising time	F	15
H - Cooking time	E & G	45

Using the above information of Illustration 1, compute Total, Free and Independent Floats for each Activity.

KEY POINTS:

Illustration-3 (OLD SM-35)

XYZ Auto-manufacturing Co. has to prepare a design of its latest model of motorcycle. The various activities tobe performed to prepare design are given in the following table: Prepare a Network diagram.

Activity	Description of activity	Preceding activity
A	Prepare drawing	—
B	Carry out cost analysis	A
C	Carry out financial analysis	A
D	Manufacture tools	C
E	Prepare bill of material	B, C
F	Receive material	D,E
G	Order sub-accessories	E
H	Receive sub-accessories	G
I	Manufacture components	F
J	Final assembly	I,H
K	Testing and shipment	J

KEY POINTS:

Illustration-4 (OLD SM-36)

A civil engineering firm has to bid for the construction of a dam. The activities and time estimates are given below:

Activity	DURATION		
	Optimistic	Most likely	Pessimistic

1—2	14	17	25
2—3	14	18	21
2—4	13	15	18
2—8	16	19	28
3—4 (dummy)			
3—5	15	18	27
4—6	13	17	21
5—7 (dummy)			
5—9	14	18	20
6—7 (dummy)			
6—8 (dummy)			
7—9	16	20	41
8—9	14	16	22

The policy of the firm with respect to submitting bids is to bid the minimum amount that will provide a 95% probability of at best breaking even. The fixed costs for the project are 8 lakhs and the variable costs are 9,000 everyday spent working on the project. The duration is in days and the costs are in terms of rupees.

What amount should the firm bid under this policy? (You may perform the calculations on duration etc. up to two decimal places.)

KEY POINTS

Illustratin-5 (CRASHING)

The following table gives data on normal time & cost and crash time & cost for a project.

Activity	Normal		Crash	
	Time (days)	Cost (Rs	Time (days)	Cost (Rs
1—2	6	600	4	1,000
1—3	4	600	2	2,000
2—4	5	500	3	1,500
2—5	3	450	1	650
3—4	6	900	4	2,000
4—6	8	800	4	3,000
	4		2	

5—6 6—7	3	400 450	2	1,000 800

The indirect cost per day is ` 100.

(i) Draw the network and identify the critical path.

(ii) What are the normal project duration and associated cost?

(iii) Crash the relevant activities systematically and determine the optimum project completion time and cost.

Illustration 39.

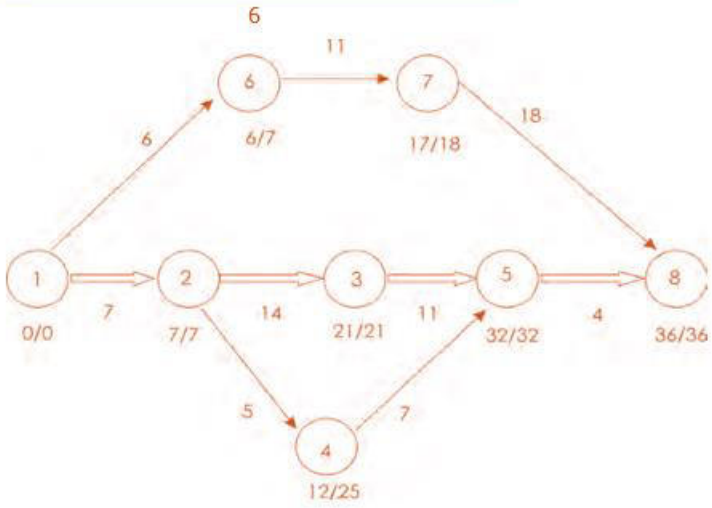
The activities involved in a PERT project are detailed in the following table:

Job(i- j)	DURATION TIME (DAYS)		
	most optimistic time	most likely time	most pessimistic time
1-2	3	6	15
2-3	6	12	30
3-5	5	11	17
7-8	4	19	28
5-8	1	4	7
6-7	3	9	27
4-5	3	6	15
1-6	2	5	14
2-4	2	5	8

Draw a network diagram

dSolution:

Calculate feasible time = $\frac{\text{Optimistic Time + 4 Likely Time + Pessimistic Time}}{6}$



Multiple Choice Questions

1. Critical Activities have
   (a) Maximum float
   (b) Minimum float
   (c) Zero float
   (d) Negative float
2. In PERT Chart, the Activity time distribution is -
   (a) Normal
   (b) Binomial
   (c) Poisson
   (d) Beta
3. A PERT Network has nine activities on its Critical Path. The Standard Deviation of each activity on the Critical Path is 3. The S. D of the Critical Path is-
   (a) 3 (b) 9 (c) 81 (d) 27
4. For an activity the pessimistic, most likely and optimistic times are respectively 10, 6 and 2 days. Theexpected duration of the activity is –
   (a) 6 days
   (b) 3 days
   (c) 2 days
   (d) 9 days
5. The time by which the activity completion time can be delayed without affecting the start of the succeedingactivities is known as –
   (a) Total float
   (b) Free float
   (c) Independent float
   (d) Head slack
6. Which of the following statement is not true?
   (a) PERT is deterministic in nature.
   (b) CPM is probabilistic in nature.
   (c) PERT Network can be crashed.
   (d) All of the above.
7. Following data refers to a project Network. What will be the Critical Path?

Activity	1 – 2	2 – 3	3 – 4	1 – 4	2 – 5	3 – 5	4 – 5
Duration	2 Days	1 Day	3 Days	3 Days	3 Days	2 Days	4 Days

   (a) 1 – 2 – 3 – 5
   (b) 1 – 2 – 3 – 4 – 5
   (c) 1 – 4 – 5
   (d) 1 – 4 – 3 – 5
8. The amount of time by which an activity can be delayed without affecting the project completion is called –
   (a) Free float
   (b) Total float
   (c) Interfering float
   (d) None of the above
9. Optimistic time and pessimistic time of an activity are respectively 4 days and 16 days. Variance of theduration of the activity will be –
   (a) 4 day2
   (b) 2 day2
   (c) 3 day2
   (d) None of the above
10. In a project planning Free float can affect which of the following?
    (a) Succeeding activity
    (b) Only that activity
    (c) Preceding activity
    (d) All of the above
11. Solution of problems of Crashing has to be started by applying the technique on –
    (a) Any activity of the Network.
    (b) Non critical activities.
    (c) Critical activities.
    (d) None of the above.
12. A PERT activity has an optimistic time of 3 days, pessimistic time of 15 days and an expected time of 7

days. What is the most likely time of the activity?

(a) 10 days (c) 5 days

(b) 6 days (d) None of the above

13. The reduction in project time normally results in –

(a) Decrease in Direct Cost and increase in Indirect Cost

(b) Increase in Direct Cost and decrease in Indirect Cost

(c) Increase in both Direct and Indirect Costs.

(d) Decrease in both Direct and Indirect Costs.

14. The Normal duration and Normal cost of an activity are respectively 10 days and Rs 350. The cost slope is Rs 75 per day. If the Crash duration is 8 days then what is the Crash cost of the activity?

(a) Rs 400/- (b) Rs 500/- (c) Rs 600/- (d) Rs 650/-

15. Which of the following is incorrect?

(a) PERT is suitable for projects having probabilistic time estimates.

(b) CPM is suitable for projects having deterministic activities.

(c) Both PERT and CPM are event oriented.

(d) PERT is event oriented while CPM is activity oriented.

16. The activity that must be completed prior to the start of an activity is called –

(a) Dummy activity (c) Concurrent activity

(b) Successor activity (d) Predecessor activity

17. The slack times of Tail and Head events of Activity P are respectively 10 days and 4 days. If the Free float of the Activity P is 12 days then the Total float would be –

(a) 8 days (c) 22 days

(b) 16 days (d) None of the above

18. Which of the following represents reduction in project duration?

(a) Crashing (c) Variance

(b) Negative slack (d) All of the above

19. Critical Path Method is good for –

(a) Small projects only (c) Both small and large projects equally

(b) Large projects only (d) Neither small nor large projects

20. The optimum duration is the –

(a) Summation of normal durations of each activity of the project.

(b) Summation of normal durations of activities in the Critical Path.

(c) One which gives the minimum Total Cost for the completion of the project.

(d) Summation of crash durations of activities in the Critical Path.

21. Which of the following is not a notable challenge while scheduling a project?

(a) Deadlines exist (c) Too many workers may be required

(b) Independent activities (d) Costly delay

22. A critical path is –

(a) The shortest path

(b) The longest path

(c) The path that begins from the start node and ends at the last node.

(d) All of the above.

23. Activities A, D and F merges at the event 6. If the earliest finish times of A, D and F are respectively 13, 17and 8 then the earliest time of Event 6 is –
    (a) 8
    (b) 13
    (c) 17
    (d) Cannot be determined from the given information.
24. Which of the following is true when a project is scheduled by Critical Path Analysis?
    (a) Work breakdown structure is used to divide the project into different activities.
    (b) Duration for each activity is established.
    (c) Precedence relationship of the activities is determined.
    (d) All of the above.
25. Total Project Cost versus Duration curve is –
    (a) Parabolic
    (b) S shaped
    (c) U shaped
    (d) Linear
26. Activities P, Q and R are the immediate successors of the activity N. If their current starting times are 10, 11and 17 respectively then what is the latest finishing time of the activity N ?
    (a) 10
    (b) 11
    (c) U shaped
    (d) None of the above
27. Activity in a Network diagram is represented by –
    (a) Circle
    (b) Rectangle
    (c) Square
    (d) Arrow
28. The particular task performance in CPM is known as –
    (a) Event
    (b) Activity
    (c) Dummy
    (d) Contract
29. Among the following, critical path and slack time analysis mostly help
    (a) Managers define the project activities
    (b) Highlight relationships among project activities.
    (c) Point out who is responsible for various activities
    (d) Pinpoint activities that need to be closely watched.

Answers:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
c	d	b	a	b	d	d	b	a	c	c	b	b	b	c
16	17	18	19	20	21	22	23	24	25	26	27	28	29	
d	b	a	b	c	b	b	c	d	c	a	d	b	d	

Learning Curve

Learning curve theorem

® When a person performs the task repeatedly, the time taken to do it gradually reduces. This reduction in time happens because of learning effect.

The learning effect occurs because:

- He becomes more familiar with the Job
- He develops better tooling methods to perform it
- He identifies and eliminates unwanted activities in performing the Job.

® Learning process will stop- after continuously repeating the job.

® Time to complete the job initially declines then stabilizes once the efficient working is achieved

® As per learning curve theorem " the cumulative average time per unit is assumed to decreases – by a constant % - every time the production doubles.

1. Cumulative average time per unit = $\frac{\text{cumulative time}}{\text{cumulative production}}$
2. Constant % = learning effect.
3. Double the production - 1,2,.4,8,16,32,64,128…….

Learning curve ratio

As per L.C theorem " each time production doubles – average time or cost decreases – by some % (learning curve ratio).

Average labor cost of first 2n units = average labor cost of first n units × L.C ratio

Learning ratio = $\frac{\text{average labour cost of first 2n units}}{\text{average labour cost of first n unuts}}$

A learning of 80% means, every time the production doubles the average time per unit becomes 80% of the previous average. Example can be understood as follows:

No. of Units	Cumulative Average time per unit (Hours)
1	100
2	80
4	64
8	51.20

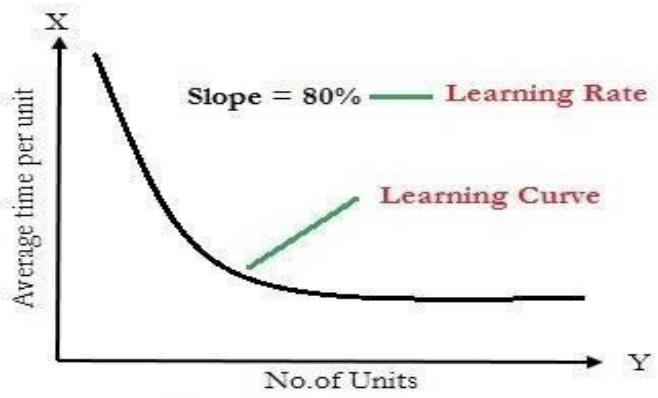
Which one is faster learning: 100% 80%, 50%, 30 %./

Meaning of 80% learning curve- time reduces by 20%(on previous avg) every time production doubles.

Assumption: Learning rate is constant over the period
May vary from person to person
But constant in industry.

Graphical presentation

The learning curve looks as follows



Learning Curve

As the number of units increases, the average time per unit decreases. The rate at which it decreases is called "Learning Rate" and is always expresses as percentage (%).

Here slope of curve defines – learning rate

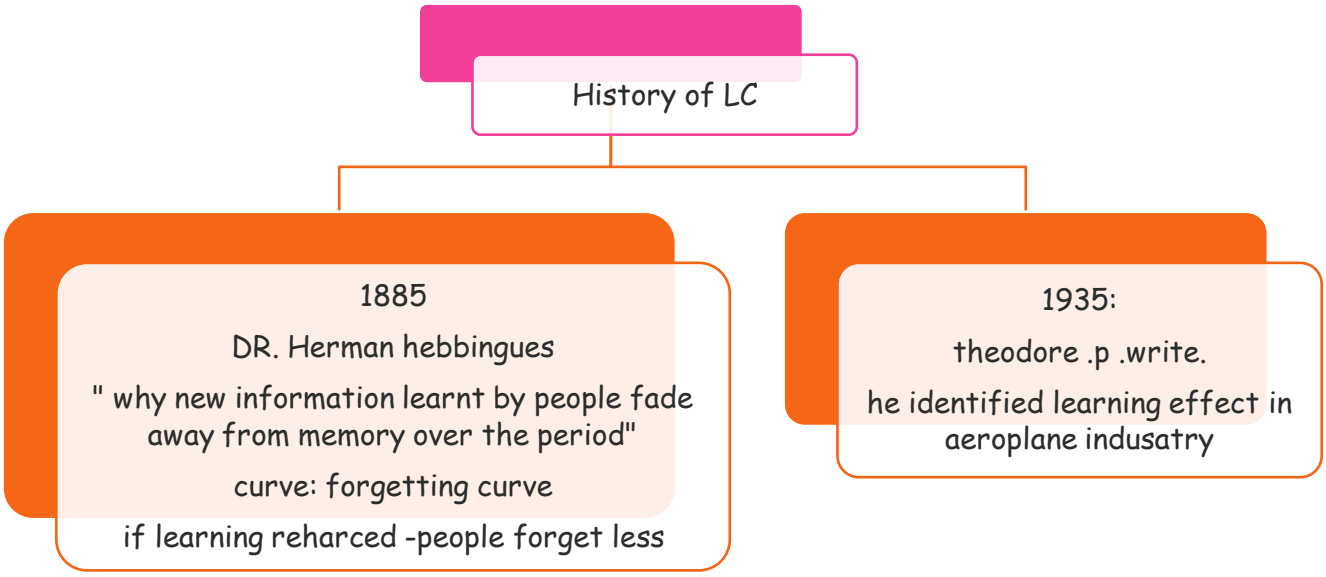
Phases in learning curve – 3 phases

1st phase:
Gradual increase in production.
Learning happens
This phase is generally STEEP

2nd phase:
learning gradually deteriorate - cause- limitation in equipment

3rd phase:
production rate decreases due to
1.customer requirement reduces
2.increase in costs

History of learning curve



Learning curve function

Theodore .p .write. in his book " factors effecting cost of aeroplanes -1936"

Defined learning curve function.

$$Y = aX^b$$

Here:

Y = average time or cost per unit.

a= time or cost per first unit.

X = cumulative volume of production.

b = learning index or learning co-efficient. $b = \frac{\log(\text{learning \%})}{\log 2}$ {percentage 80%- in decimals 0.80}

Important points

1) The other names given to learning curve are **"Experience Curve" "Improvement Curve"** and "Progress Curve".
2) The learning curve can be applied by the management accountant in the following three areas:
   a) Price fixation
   b) Fixing Labor Standards for Variance analysis
   c) Volume determination for a given capacity
3) Learning curve will not have impact in following situations:
   a) Where the production is automated involving lesser human element
   b) Where the Jobs are non-repetitive or User Specific or Customer Specific

   Where the job is performed by highly experienced persons who has reached saturation pointor a state of steady point

Some basic logarithm formulae:

Decimal function	Logarithm function
$A \times B$	$\log A + \log B$
$\frac{A}{B}$	$\log A - \log B$
A^B	$B \log A$
A+B	$\log(A + B)$
A-B	$\log(A - B)$
$\log_A^A$	1

How to find log value on simple calculator:

S-1 : take log value on simple calculator

S-2 : press $\sqrt{\ }$ 15 times.

S-3 : subtract 1 (-1)

S-4 : finally subtract it with 0.000070271.

Try ex : log 2 = 0.3010.

Log 0,80 =0.0969.

How to find anti log on simple calculator:

S-1: Take Anti-log value on simple calculator

S-2: then type $\div 227695 + 1 =$

S-3: press $\times =$ 19 times

Try ex : anti log 0.5811 = ?

How to find power (exponential) value on simple calculator:

S-1: Take base value press $\sqrt{\ }$ 14 times

S-2: -1

S-3: $\times$ power value

S-4: + 1

S-5: $\times = 14$ times

Try ex: $40^{-0.322}$ = $(\frac{1}{40})^{322}$

Illustration-1

The usual Learning Curve model is $Y = ax^b$ where
Y is the average time per unit for x units and 'a' is the time for first unit x is the cumulative number of units

b is the learning coefficient and is equal to (log 0.8)/(log 2) = –0.322 for a learning rate of 80%

Given that a = 10 hours, you are required to Calculate:

(i) The average time for 20 units.
(ii) The total time for 30 units.
(iii) The time for units 31 to 40.

Given that log 2 = 0.301, Antilog of 0.5811 = 3.812
log 3 = 0.4771, Antilog of 0.5244 = 3.345.
log 4 = 0.6021, Antilog of 0.4841 = 3.049.

Key notes

Illustration-2

The Learning Curve in management accounting has now become or is going to become an accepted tool in industry, for its applications are almost unlimited. When it is used correctly, it can lead to increased business and higher profits; when used without proper knowledge, it can lead to lost business and bankruptcy. State precisely:

(i) Your understanding of the Learning Curve:
(ii) The theory of Learning Curve;
(iii) The areas where Learning Curves may assist in management accounting; and
(iv) Illustrate the use of Learning Curves for calculating the expected average unit cost of making-
(a) 4 machines (b) 8 machines

Using the data below:

Data:

Direct Labour needed to make first machine = 1000 hrs. Learning Curve = 90%
Direct Labour cost = Rs 15 per hour.
Direct materials cost = Rs 1,50,000
Fixed cost for either size orders = Rs 60,000.

Ans :

Areas of consequence:

(i) A Standard Costing system would need to set standard labour times after the

learning curve had reached a plateau.

(ii) A budget will need to incorporate a learning cost factor until the plateau is reached.

(iii) A budgetary control system incorporating labour variances will have to make allowances for the anticipated time changes.

(iv) Identification of the learning curve will permit the company to better plan its marketing, work scheduling, recruitment and material acquisition activities. As the employees gain experience they are more likely to reduce material wastage

Key notes:

Illustration-3

Z.P.L.C experiences difficulty in its budgeting process because it finds it necessary to quantify the learning effect as new products are introduced.

Substantial product changes occur and result in the need for retraining.

An order for 30 units of a new product has been received by Z.P.L.C So far, 14 have been completed; the first unitrequired 40 direct labour hours and a total of 240 direct labour has been recorded for the 14 units.

The productionmanager expects an 80% learning effect for this type of work.

The company uses standard absorption costing. The costs attributed to the centre in which the unit is manufacturedare as follows:

Head	Cost
Direct material	Rs 30.00 per unit.
Direct Labour	Rs 6.00 per hour.
Variable overhead	Rs 0.50 per direct labour hour.
Fixed overhead	Rs 6,000 per 4 week operating period.

There are ten direct employees working a five-day week, eight hours per day. Personal and other downtimeallowances account for 25% of total available time.

The company usually quotes a four-week delivery period for orders. You are required to:

Determine whether the assumption of an 80% learning effect is a reasonable one in this case, by using thestandard formula $Y = ax^b$

Where Y = the cumulative average direct labour time per unit (productivity)a = the average labour time per unit for the first batch.

x = the cumulative number of batches produced. b = the index of learning.

(i) Calculate the number of direct labour hours likely to be required for an expected second order of 20 units.

(ii) Use the cost data given to produce an estimated product cost for the initial order, examine the problemswhich may be created for budgeting by the presence of the learning effect.

Use logarithmic tables to find the values of Logarithm and Anti-Logarithm.

Key notes:

Illustration-4

A firm received an order to make and supply eight units of standard product which involves intricate labour operations. The first unit was made in 10 hours. It is understood that this type of operation is subject to an 80% learning rate. The workers are getting wages at the rate of Rs12 per hour.

(i) What is the total time and labour cost required to execute the above order?

(ii) If a repeat order of 24 units is also received from the same customer, what is the labour cost necessary for the second order?

KEY NOTES:

Multiple Choice Questions

1. A Learning Curve describes
   - (a) The increase in number of units produced per unit time as the total number of units produced increases
   - (b) The rate at which an organisation acquires new information.
   - (c) The amount of production time per unit as the total number of units produced increases.
   - (d) The increase in production time as the total number of units produced increases.
2. Limitations of the Learning Curve approach include –
   - (a) Learning Curves must be redeveloped whenever the product or the production process is modified.
   - (b) Learning Curves are applicable when considering a highly automated process.
   - (c) Learning Curves are only valid when considering simple production process.
   - (d) Learning Curves are only valid when the total number of units produced is relatively small.
3. Which of the following statements about Learning Curve is incorrect?
   - (a) A change in the process disrupts the Learning Curve.
   - (b) The rate of learning varies depending on the quality of management.
   - (c) The Learning Curve can be disrupted by the change in personnel.
   - (d) Learning Curves show that the time saved in completing each subsequent unit increases.

4. Which of the following is not an application of Learning Curve?
   (a) Learning Curves allow a manager to predict the time required for new employee orientation on company policies and procedures.
   (b) Learning Curves permit a manager to prepare a work schedule.
   (c) Learning Curves allow a manager to forecast the labour requirements while preparing a departmental employee budget.
   (d) Learning Curves can be employed in supply chain negotiations.
5. The Learning Curve remains valid –
   (a) When applied to different firms in the same industry.
   (b) For product modification that will simplify the product assembly.
   (c) As long as process revisions involve only the addition of automated machinery.
   (d) As long as production volume increases and processes are consistent.
6. A 100% Learning Curve implies that –
   (a) Organisational learning has taken place.
   (b) The Learning Curve can be applied universally across an industry.
   (c) No learning has been achieved.
   (d) Direct labour time is reduced 100% for each doubling of production.
7. Which of the following statement is true?
   (a) The Learning Curve displays the relationship between process time per unit and the cumulative number of units produced.
   (b) Learning Curves are most easily developed for one off type orders.
   (c) Learning Curves are based on the Normal distribution.
   (d) The standard time for a process is determined from the Learning Curve when cumulative production isfifty units.
8. Learning Curve is also known as –
   (a) Growth curve
   (b) Production curve
   (c) Exponential curve
   (d) Experience curve
9. For organisations with wide variety of product range which of the following statement is correct?
   (a) Different product will have different Learning Curves.
   (b) Learning percentage for different product will be different.
   (c) Both (a) and (b)
   (d) None of the above

Answers:

1	2	3	4	5	6	7	8	9
c	b	d	d	d	c	a	d	c

CONCEPT - 1 Some important terminology related to the concept of Profit maximization

Revenue:

- The Revenue is defined as the money earned by selling certain quantity of output.
- Revenue involves three inter-related concepts – Total Revenue (TR), Average Revenue (AR) & Marginal Revenue (MR).
- Total Revenue is the product of Price (P) and the Quantity Sold (x). Thus TR = P.x
- Average Revenue is the Revenue earned per unit sale.
  So AR = TR / x (Or), AR = Px /x Or, AR = P = Price
- Hence Average Revenue can be termed as the Price or vice versa. Thus Average Revenue curve is the same as the Demand curve.
- Marginal Revenue is defined as the revenue earned by selling an additional unit of output.
- MR is the rate of change of Total Revenue with respect to Output which is nothing but the derivative of Total Revenue with respect to Output. Hence MR = $\frac{d}{dx}$ (TR)
- Mathematically Revenue is expressed as a function of Output. That is R = f(x)

Cost of Production:

- Cost, too has three basic concepts – Total Cost (TC), Average Cost (AC) and Marginal Cost (MC)
- Total Cost has two components: Fixed and Variable. So TC = TFC + TVC
- Average Cost is given as AC = TC/ x (Or), AC = TFC/ x + TVC/ x (Or), AC = AFC + AVC
- Marginal Cost is the cost of producing an additional unit of output.
- Mathematically, MC = $\frac{d}{dx}$ (TC) = $\frac{d}{dx}$ (TFC + TVC)
- Cost of Production is expressed as a function of Output in order to know its behavior at different levels of output produced or capacity utilization. So C = f(x) is the mathematical way of representation of Cost.

Profit:

- It is the residual after deducting Cost from the Revenue.
  It is represented as π = f(x)
- Total Profit = T π = TR – TC Or, T π = TR – TC Or, Average Profit = A π = T π/x Or,
  A π = AR – AC
- Marginal Profit = M π = $\frac{d}{dx}$ (Tπ) or, M π = $\frac{d}{dx}$ (TR) - $\frac{d}{dx}$ (TC) Or,
  M π = MR – MC

Concept – 1.1 Break-even Point:

® Break-even point is the point at which total cost and total revenue are equal which means that the firm does neither earn any profit nor any loss. The firm faces a no gain no loss situation. Thus, at the *Break – even Point* :

Profit = 0

Total Cost = Total Revenue.

CONCEPT 1.2 :Review of Standard formulae and rules of Differentiation

0. $\frac{d}{dx}f(x) = \frac{f(x+h)-f(x)}{h}$	
1. $\frac{d}{dx}\,x^n = n\,x^{n-1}$	2. $\frac{d}{dx}\left(\sqrt{x}\right) = \frac{1}{2\sqrt{x}}$
3. $\frac{d}{dx}e^x = e^x$. where e = constant = 2.718(approx.)	4. $\frac{d}{dx}a^x = a^x log_e^a$
5. $\frac{d}{dx}\left(\frac{1}{x^n}\right) = -\frac{n}{x^{n-1}}$.	6. $\frac{d}{dx}(log_e^x) = \frac{1}{x}$, $\frac{d}{dx}(log_a^x) = \frac{1}{x}log_a^e$
7. $\frac{d}{dx}(c) = 0$	8. $\frac{d}{dx}(u \pm v) = \frac{du}{dx} \pm \frac{dv}{dx}$
9. $\frac{d}{dx}(uv) = \frac{du}{dx}\,v + \frac{dv}{dx}\,u$	10. $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{\frac{du}{dx}v - \frac{dv}{dx}u}{x^2}$
11. If y = f(x) =k then $\frac{dy}{dx}$ = f' (x)= 0, where k is a constant	
12. If y = f(x) = p(x) ± q(x) then $\frac{dy}{dx}$ =f'(x) = p'(x) ± q'(x)	
13. If y = f(x) = e^{mx} then $\frac{dy}{dx}$ = f'(x) = me^{mx}, where 'm' is a constant.	

CONCEPT 1.3 : Partial differentiation

In case of Partial differentiation, all the above formulae / rules hold good . but the exception that all the other variables present in the given function are considered as constant except the ones with the participating variable of partial differentiation.

Some examples are -

a) If U = xy then $\frac{\partial U}{\partial x}$ = y and $\frac{\partial U}{\partial y}$ = x

b) If U = $x^a y^b$ then $\frac{\partial U}{\partial x}$ = $ax^{a-1}y^b$ and ∂U/∂y = $bx^a y^{b-1}$ [where 'a' and 'b' are constants]

c) If U = px + qy then ∂U/∂x = p and ∂U/∂y = q [where 'a' and 'b' are constants]

d) If U = a√x + b√y then ∂U/∂x = a/2.$(x)^{-1/2}$ and ∂U/∂y = b/2.$(y)^{-1/2}$[where 'a' and 'b' are constants]

CONCEPT – 2 : OPTIMISATION

There can be two different situations under which the optimization of objectives are carried out. These are given as – (1) Unconstrained Optimization and (2) Constrained Optimization

Unconstrained Optimization can further have sub divisions as given below -

(A) Optimization of single variable objective function
(B) Optimization of objective functions having multiple variables.

Similarly Constrained Optimization has the following sub divisions.

(A) Equality constrained Optimization
(B) Inequality constrained Optimization
(C) Static Optimization
(D) Dynamic Optimization

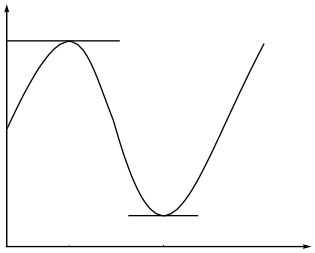
Differential calculus is used a lot for study of the optimization in Economics.. The standard formulae and rules are given below.

CONCEPT 2.1 : UNCONSTRAINED OPTIMIZATION

A major part of economic analysis assumes not only maximizing behavior on the part of the economic factors but also unconstrained optimization or mathematical optimization.
Such type of optimization is also known as Unbounded Maxima technique.
There can be two different situations involving either single variable or multiple variables.

2.1.A : Optimization of Functions involving SINGLE INDEPENDENT VARIABLE

When the objective function is given as an Algebraic Function and no constraints are imposed then to check weather the function is maximum or minimum



Step - 1 : Find the first derivative
Step – 2 : Make first derivative = 0 $\frac{dy}{dx} = 0$ (we will get x values)
Step – 3 : Find the second derivative
If second derivative – positive : f(X) is minimum
If second derivative – negative : f(X) is maximum
If $\frac{d^2y}{dx^2} = 0$ then there exist neither a maxima nor a minima. Such a point is known as **Point of Inflexion.**

2.1.B : Optimization of Functions involving MULTIPLE INDEPENDENT VARIABLES.

Objective Function involves more than one Independent variable (say two) and the change in the dependent variable is the joint impact of changes in both the variables
The measurement of the independent impact of one variable is not possible without assuming that the other variable remains unchanged. Partial Derivative of a function explains the same logic mathematically.

The conditions of optimization in this case are given as follows -

Step – 1 : 1st Order Partial Derivatives.

- $\frac{\partial f}{\partial x}$ or f_x and $\frac{\partial f}{\partial x}$ or f_y

Step – 2 : The values of the 1st Order Partial Derivatives should be Zero.

- $\frac{\partial f}{\partial x}$ or $f_x = 0$ and $\frac{\partial f}{\partial x}$ or $f_y = 0$,
- From the above equations the critical values of x and y are determined.
- The coordinates of the critical point (a,b) are determined, where a= value of x and b = value of y for a **bivariate function**.

Step – 3 : 2nd Order Partial Derivatives

- These are $\frac{\partial^2 f}{\partial x^2}$ or f_{xx}, $\frac{\partial^2 f}{\partial y}$ or f_{yy}, $\frac{\partial^2 f}{\partial y \partial x}$ or f_{xy} and $\frac{\partial^2 f}{\partial x \partial y}$ or f_{yx} .
- Calculate the numerical values of these by putting x= a and y= b

Assume:- $A = f_{xx}(a,b)$, $B = f_{xy}(a,b)$ and $C = f_{xx}$ and find the value of $D = AC - B^2$

- If $D > 0$ and $A, C > 0$ then there is a **local Minima at (a,b)**
- If $D > 0$ and $A, C < 0$ then there is a **local Maxima at (a,b)**
- If $D < 0$ then **(a,b) is a Saddle Point**
- If $D = 0$ then **the test fails.**

[**Note:** The expression $D = AC - B^2$ can also be represented in the form of a determinant as follows –

$$D = \begin{vmatrix} A & B \\ B & C \end{vmatrix} \text{ or, } D = \begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix}.$$

Note :

- As f_{xy} is equal to f_{yx} always, we represent both as B. This determinant actually corresponds to a (2 × 2) Matrix called **Hessian Matrix.**
- So Hessian Matrix of order 2 is given as $\begin{bmatrix} I_{xx} & I_{xy} \\ f_{yx} & f_{yy} \end{bmatrix}$
- And the same is applicable when there are two variables in the problem. For problems with three variables, we will comeacross Hessian Matrix of order 3.]

EQUILIBRIUM OF A FIRM

A firm is in equilibrium when, given the demand and cost conditions, it produces that level of output at which the Profit is maximized.

Neo - Classical theory of Economics assumes maximization of Profit is the sole objective of it. The level of output and the price charged corresponding to the equilibrium are called the Equilibrium Output and the Equilibrium Price respectively.

1. Condition for Firm's Equilibrium

It has already been discussed that a Firm is said to be in Equilibrium when it maximizes its Profit. i.e. marginal profit = 0

$$\frac{d}{dx}(T\pi) = 0 \text{ and } \frac{d^2}{dx^2}(T\pi) < 0$$

Now, $T\pi = TR - TC$

Differentiating both sides with respect to x we get

$$\frac{d}{dx}(T\pi) = \frac{d}{dx}(TR) - \frac{d}{dx}(TC) \text{ (or) ,}$$

$$\frac{d}{dx}(T\pi) = MR - MC$$

But,

$(T\pi) = 0$ Or, $MR - MC = 0$ Or,

MR = MC is the Condition for a Firm's Equilibrium

This is the **necessary condition** of a Firm's Equilibrium.

For the sufficient condition of the Firm's Equilibrium we should have

$$\frac{d^2}{dx^2}(T\pi) < 0 \quad \text{Or,}$$

$$\frac{d^2}{dx^2}(TR - TC) < 0 \quad \text{Or}$$

$$, \frac{d^2}{dx^2}(TR) - \frac{d^2}{dx^2}(TC) < 0 \quad \text{Or}$$

$$\frac{d}{dx}[MR] < \frac{d}{dx}[MC]$$

The conditions written above are meant for single variable type situations of optimization.

In case the number of variables is more than one then the Condition of Firm's Equilibrium for the following situations are –

1. For Multi-plant Monopolist Firm producing the same product in all the Plants

condition : **MR = MC_1 = MC_2,**

where MC_1 and MC_2 = Respective Marginal Costs of Productionin Plants 1 & 2 .

The checking condition is given as

$\frac{\partial^2}{\partial x^2}(TR) < \frac{\partial^2}{\partial x_1^2}(TC_1)$ and

$\frac{\partial^2}{\partial x^2}(TR) < \frac{\partial^2}{\partial x_2^2}(TC_2)$

[x_1 & x_2 are the quantities produced in the plants 1 and 2 respectively. Also $x = x_1 + x_2$]

2. For Price discriminating Monopolist Firm selling the same product at different prices in different markets

condition : **MC = MR_1 = MR_2,**

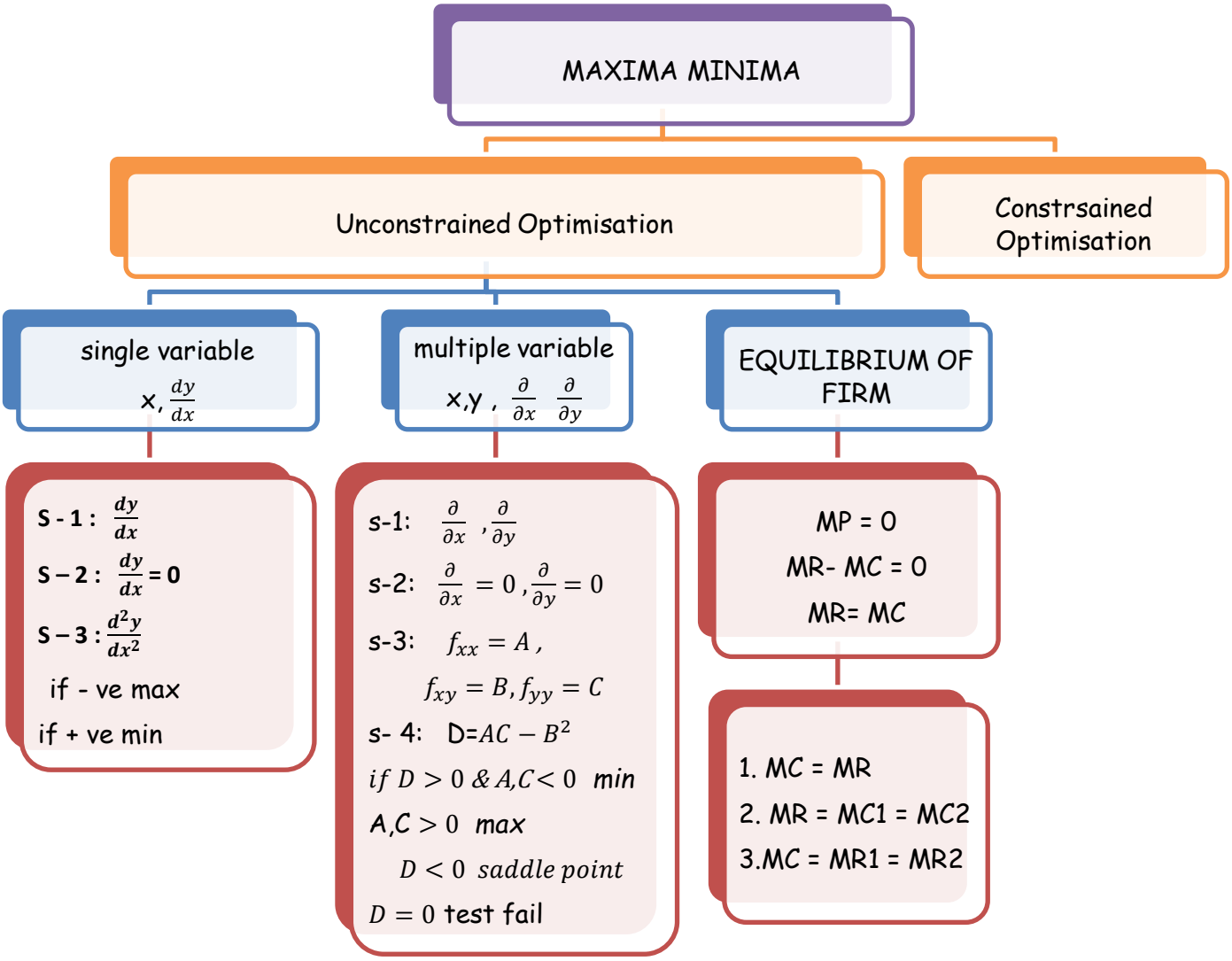
where $MR_1$and MR_2 are the respective Marginal Revenues for the two different market segments having Demand Functions given as p_1 and p_2

The **checking condition** is given as

$\frac{\partial^2}{\partial x_1^2}(TR_1) < \frac{\partial^2}{\partial x^2}(TC)$ **and**

$\frac{\partial^2}{\partial x_2^2}(TR_2) < \frac{\partial^2}{\partial x^2}(TC)$

[x_1 & x_2 are the quantities sold at the prices p_1 and p_2 respectively. Also $x = x_1 + x_2$] .


MAXIMA MINIMA
Unconstrained Optimisation
Constrsained Optimisation
single variable
x, $\frac{dy}{dx}$
multiple variable
x,y , $\frac{\partial}{\partial x}$ $\frac{\partial}{\partial y}$
EQUILIBRIUM OF FIRM
S - 1 : $\frac{dy}{dx}$
S – 2 : $\frac{dy}{dx} = 0$
S – 3 : $\frac{d^2y}{dx^2}$
if - ve max
if + ve min
s-1: $\frac{\partial}{\partial x}$, $\frac{\partial}{\partial y}$
s-2: $\frac{\partial}{\partial x} = 0$, $\frac{\partial}{\partial y} = 0$
s-3: $f_{xx} = A$,
$f_{xy} = B, f_{yy} = C$
s- 4: D=$AC - B^2$
$if\ D > 0\ \&\ A,C < 0$ min
$A,C > 0$ max
$D < 0$ saddle point
$D = 0$ test fail
MP = 0
MR- MC = 0
MR= MC
1. MC = MR
2. MR = MC1 = MC2
3.MC = MR1 = MR2

CONCEPT 2.2 : CONSTRAINED OPTIMIZATION

Under above Unbounded Maxima/ un constrained optimization

- It assumed that the Firm is capable of producing without any restrictions.
- All the resources required to produce the given level of output are unlimited .
- It has no shortage of inputs, energy,labour, transport, liquidity etc.

But under bounded Maxima/ constrained optimization

- practical life the conditions are not exactly like as in case of Unbounded Maxima.
- There is a limitation on availability of resources (men ,material ,money machine ,,).
- Optimization with restrictions of different kind is known as Constrained Optimization.

Mathematically the constraints are expressed either in the form of Equations (which are also known as Equalities)or in the form of Inequalities.

Problems of equality constraints are dealt with **LAGRANGIAN MULTIPLIERS** and those involving inequality constraints are solved by **LINEAR PROGRAMMING TECHNIQUES**.

Optimization with SINGLE EQUALITY CONSTRAINT

Ex : A consumer has a choice of two commodities X & Y. Prices of these are p and q respectively. The person haslimited money (M) and wants to procure maximum possible quantities of X & Y with the amount he / she has.

The problem can be rewritten as –

Maximize Utility function U = f (X, Y) subject to the constraint PX + QY = M

STEPS to be followed by Lagrangian Multiplier Method are –

1. Transform the Constraint equation to a form with 0 on the R.H.S. In this case it is
   PX + QY – M = 0
2. Multiply L.H.S of the transformed Constraint Equation by Lagrange's Multiplier (λ). In this case it takes the form **λ.(pX + qY – M)**
3. From the original Objective Function ,subtract the one obtained in the previous step to form Lagrangian function
   L(X, Y, λ) = f(X,Y) - λ.(pX + qY – M)
   [It can be noted that L(X, Y, λ) = f(X,Y) when the constraint holds i.e. pX + qY = M. Hence maximization of Lagrangian function ultimately maximizes the original objective function f(X,Y)]
4. Find the critical values of the unknowns (X, Y and λ) using the 1st Order conditions i.e. all the partial derivatives are equal to zero.
   Thus, $\partial L/\partial X = 0$ Or, $f_X - \lambda p = 0$
   $\partial L/\partial Y = 0$ Or, $f_Y - \lambda q = 0$ and
   $\partial L/\partial \lambda = 0$ Or, $pX + qY - M = 0$
   Solving the above three equations one can get the values of three unknowns X, Y and λ.
5. Now find a Bordered Hessian Matrix (HB) given as –
   $$HB = \begin{bmatrix} 0 & g_x & g_y \\ g_x & L_{xx} & L_{xy} \\ g_y & L_{yx} & L_{yy} \end{bmatrix}$$
   It can be noted that above are the partial derivatives with respect to x and y respectively for the function given by g(X,Y) = pX + qY – M i.e. the given constraint expressed as a function.

6. Find the value of the Determinant corresponding to the matrix HB i.e. evaluate Det. HB
7. If **Det.HB > 0** then the critical values of X and Y obtained in step (4) corresponds to a **Maxima.**
   If **Det.HB < 0** then the critical values of X and Y obtained in step (4) corresponds to a **Minima.**

Illustration-1(Maximization problem)
The demand (rides per day) of Roller Coaster Ride in an Entertainment Park in one of the metro cities is given by the equation $q = -450p + 41500$, where p = Price per ride in ` What price should have been charged to maximize the Total Revenue?

Illustration-2 (Minimization problem)
Assume the Cost (`) of manufacturing x numbers of a product per day is $C(x) = 14400 + 550x + 0.01x^2$. How many of the product should be manufactured per day so that the Average Cost is minimum? Also find the values of the Average Cost and the Total Cost at this level of production.

Illustration-3 (Maximization problem)
A company produces two products x and y. The total Profit (in ` '000) earned by the company is expressed algebraically by the function $P = 100x - x^2 - 2xy + 200y - 3y^2$. Find the Profit maximizing quantities of theproducts. Also find out the maximum Profit.

Illustration-4
Find the Critical Points for the function $f(x,y) = x^2y - 2xy^2 + 3xy + 4$. Examine the presence of Local Maximaand Minima among the Critical Points.

Illustration-5
A firm has the Cost function $C = x^3/3 - 7x^2 + 111x + 50$ and Demand function $x = 100 - p$. Determine theEquilibrium Output, Price and Profit earned.

Illustration-6
A manufacturer produces a liquid commodity at two different Plants located in the two regions of the country. The selling price (in ` /litre) of the product is given by the equation $p = 200 - 0.8x$, where $x = x_1 + x_2$ = Total production of the two Plants together. The Cost functions of the 2 Plants are given as $C = 0.3(x)^2 + 60x + 5000$ and $c_2 = 0.5(x_2)^2 + 30x_2 + 8000$. Determine the quantities produced by the two Plants which will put the manufacturer into an equilibrium condition.

Illustration-7
A discriminating Monopolist is able to separate its customers into two markets with respective Demand functionsas $x_1 = 16 - 0.2p_1$ and $x_2 = 9 - 0.05p_2$. Total Cost function is $TC = 20 + 20x$, where $x = x_1 + x_2$. Determine the Equilibrium Price of the product in the two markets. Also determine the Equilibrium Profit.

Illustration-8
Suppose a firm produces TV Sets at two different locations which produced x_1 and x_2 sets respectively. The joint cost function is given as $C = 0.1(x)^2 + 0.2(x)^2 + 0.2x\,x + 180x + 60x + 25000$. If the firm has to supply 1000 sets of TV then find the number of sets to be produced in the two plants so that the joint cost is minimum.

Multiple Choice Questions

1. Optimization is the method of finding
   (a) The maximum point (c) The critical point
   (b) The minimum point (d) All of the above
2. Choose the correct answer
   (a) Optimization problems should have only one objective function
   (b) Constraint functions are compulsory for any optimization problem.
   (c) Objective function must be a continuous function
   (d) None of the above
3. The process of finding relative maximum or minimum of a function is known as
   (a) Optimization (c) Minimization
   (b) Maximization (d) Any of these
4. For a Cost Function $TC = 3Q^2 + 7Q + 12$, MC is –
   (a) 6Q (c) 3Q+12
   (b) 6Q + 7 (d) None of the above
5. MR is
   (a) First order derivative of TC (c) First order derivative of TR
   (b) Second order derivative of TR (d) Second order derivative of TC
6. In unconstrained optimization with single variable the sufficient condition for maximization is –
   (a) Second order derivative of the objective function must be zero.
   (b) Second order derivative of the objective function must be less than zero
   (c) Second order derivative of the objective function must be less than zero.
   (d) None of the above
7. In case of unconstrained optimization involving two variables the necessary condition is –
   (a) First order derivative of the objective function with respect to the variables should be zero.
   (b) First order partial derivative of the objective function with respect to the variables should be zero.
   (c) Either one of (a) and (b)
   (d) Both (a) and (b)
8. A Firm is said to achieve Condition of equilibrium when
   (a) Its objective is optimized. (c) Its loss is minimized
   (b) Its profit is maximized. (d) All of the above.
9. In the expression $D = AC - B^2$ used for describing the sufficient conditions for unconstrained optimizationinvolving two variables (x and t), the meaning of A and C are –
   (a) 2nd order partial derivative of the objective function (f) with respect to x and y respectively.
   (b) 2nd order partial derivative of $\partial f/\partial x$ with respect to y
   (c) Both (a) and (b)
   (d) Only (a) but not (b)
10. A price discriminating Monopolist Firm operates in –
    (a) Such a Market where it is the sole supplier.
    (b) More than one Market.
    (c) Markets where it sells same product but in different prices.
    (d) All of the above.

11. In the expression $D = AC - B^2$ used for describing the sufficient conditions for a dual variable unconstrained optimization the term D is known as –
    (a) Hessian Matrix of order 2
    (b) Determinant for Hessian Matrix of order 2.
    (c) Matrix of partial derivatives of order 2.
    (d) Determinant of the Matrix of partial derivatives
12. For a dual plant Monopolist Firm with respective production costs C_1 & C_2 in the two plants, the necessary condition of equilibrium is
    (a) $MC_1 = MC_2 \neq MR$
    (b) $MC = MR$
    (c) $MC_1 = MC_2 = MR$
    (d) $MC_1 = MR_1$ & $MC_2 = MR_2$
13. Use of Lagrange's Multiplier is seen while –
    (a) Solving a problem of unconstrained optimization with single variable.
    (b) Solving a problem of optimization with inequality constraints.
    (c) Solving a problem of optimization with one equality constraint.
    (d) Solving a problem of optimization having no constraint.
14. For a firm the total cost function is $C(x) = -0.5x^2 + 11x + 600$. Which of the following statement is incorrect?
    (a) Average variable cost function is $AVC(x) = -0.5x + 11$
    (b) Marginal cost function is $MC(x) = -x + 11$
    (c) Cost of producing 10 units is ` 710/-
    (d) Average cost function is $AC(x) = -0.5x + 11 + 600/x$
15. Which one of the following statement is not correct?
    (a) Average Revenue of a Firm is same as the price at which its product is sold.
    (b) Total Profit is the product of quantity sold and the difference of Average Revenue and Average Cost.
    (c) When Marginal Revenue is zero then Total Revenue is maximum
    (d) None of the above.

Answers:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
d	a	a	b	c	b	b	d	d	d	b	c	c	c	d

CONCEPT – 1 : FORECASTING

Forecasting is a technique of a business's future developments based on trends, patterns and analysis of data – both current and historical. An accurate business forecast helps to create business budgets, allocate funds, make decisions about cash flow and credit needs and to create timelines for new initiatives or acquisitions..

Importance of Business Forecasting in different areas of an organisation

Business forecasting is one of the business tools been used by organisations towards future projections.Its importance is listed in the following areas:

- For a manufacturing entity to schedule its production – when to produce at full capacityand when not to.
- Helps purchasing and supply department towards planning and procurement of raw materials as well asscheduling their delivery time.
- Helps human resources department to plan the manpower requirement as the time it is required
- Facilitates allocation of resources among the functional areas of the organisation as well as control theoperations of the entity.
- Useful for cost reduction and profit increase. The management can stick to an accurate budget based oncurrent market conditions and expected future outcomes.
- Helps to foresee market trends causing the management to take measures proactively.

Types of Business Forecasts

There can be various types of Business forecasts depending on what the company wants to know or predict.

1. **General Business Forecasting** - This is used for determining overall marketconditions and the impact of the environmental factors in which the business operates.
2. **Financial Forecasting**. - It includes weighing assets and liabilities, accounts payable and receivable, operating costs, capital structure and cash flow and general market conditions. T
   his is bestsuited for any business looking to stay on top of its business's health through financial projections.
3. **Accounting Forecast** - This is used for determining the future operating costs of a business and suitable for any business concerned with covering future costs.
4. **Demand Forecasting** - Demand forecast will predict what the marketneeds and the sales forecast will predict how a business will be able to capitalize with those needs.
5. **Sales Forecasting** - is used to estimate future sales of a specific product or service within the rage of offerings of a business, using the available sales data. It allows the management to anticipate the future needs of workforce, resources, cash flow, inventory and capital investment.
6. **Capital Forecasting** is based on current and future assets and liabilities as well as predictions for liquid capital and cash flow estimates..

CONCEPT – 2 Methods or models of Business Forecasting

There are several forecasting methods or models, they all fall within two general categories – QUANTITATIVE and QUALITATIVE.
Quantitative Forecasting focuses on structured data, statistical analysisand experiments while Qualitative Forecasting uses unstructured data since it relies on interviews, surveys and observations.

CONCEPT – 3 : Quantitative Methods of Forecasting

As the name implies Quantitative Method of Forecasting is all about numbers and measureable data.

Some of the most frequently used Quantitative Methods of Forecasting are –

a. **Regression Analysis**
b. **Time Series Analysis**
c. **Exponential Smoothing**
d. **Input – Output Models**

CONCEPT – 4: Qualitative Methods of Forecasting

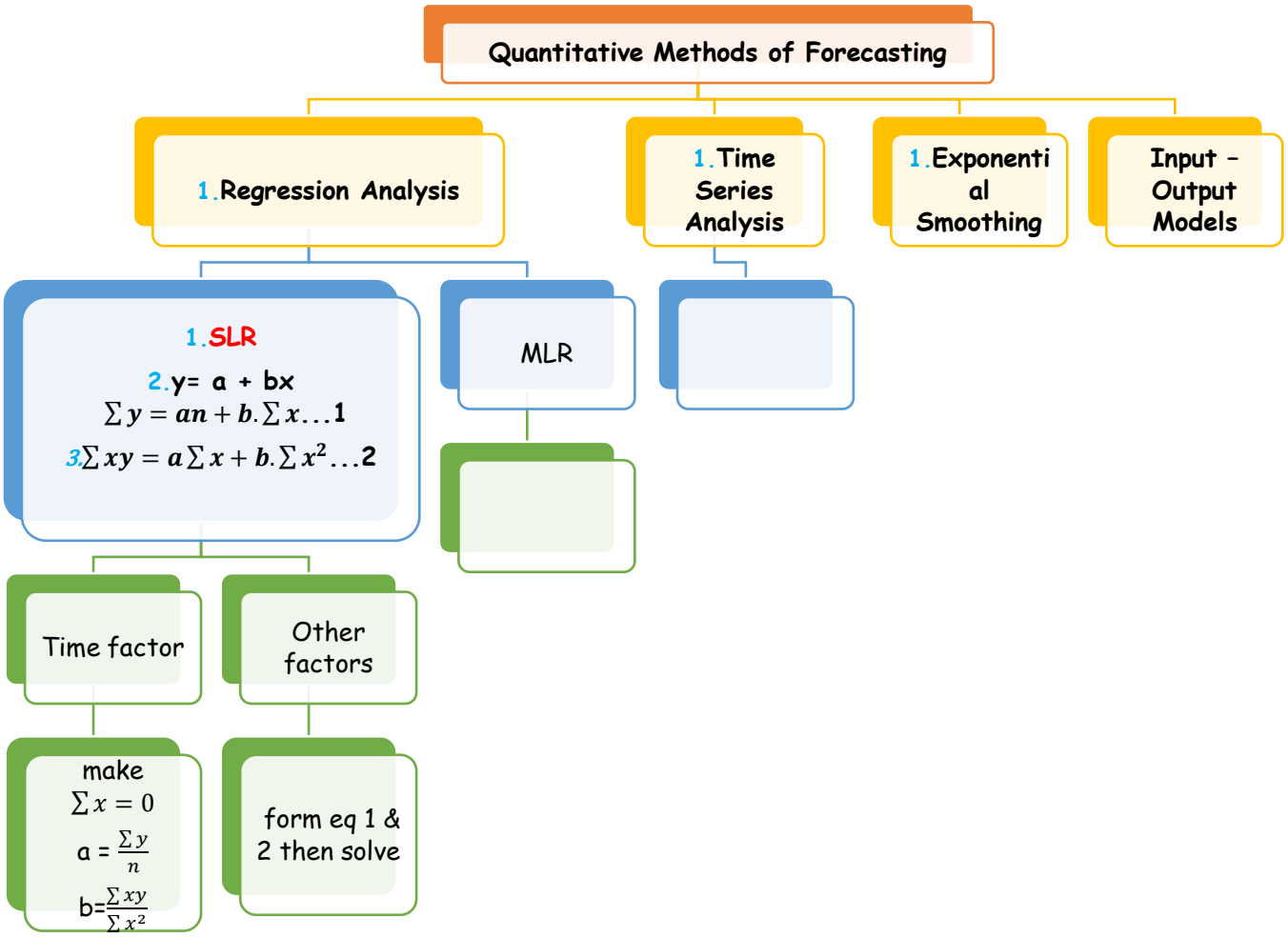
These methods aim to gain a qualitative understanding of a given subject, problem or point of interest.

Qualitative forecasting focuses on human feedback from experts and customers.

In Qualitative forecasting there are useful techniques given as –

a. **Historical Analogy Method**
b. **Executive Opinion Method**
c. **Survey Techniques**
d. **Barometric Techniques**
e. **Delphi Technique**

CONCEPT – 4 : Qualitative Methods of Forecasting	
Historical Analogy Method	Under this method forecast is based on some analogous conditions elsewhere in the past. For example, the forecast of demand for a product in India can be based on the demand for the same in some developed country in the past if it is found that the present conditions in India are very much like the same prevailing in that country in the past. But it is difficult to quantify most of these phenomena.
Jury or Executive Opinion Method	This method attempts to pool the knowledge, experience and judgement of managers inside the organisation, by asking about their opinion on the likely sales of a product in future. This is particularly applicable for a new product which does not have and past data to facilitate forecasting.
Survey Technique	Data on the certain desired variables can be obtained through selective surveys through questionnaires and interviews. The data so collected can be processed and analysed for purposes making predictions and future estimates.
Barometric Technique	The behaviour of certain economic or business variables can have an important effect on some other variables. Eg. a shortfall in supply of a commodity may lead to a rise in its price. Here the shortfall is an indicator or a barometer of the likely increase in price. Indicators can be of three types – Lead, Lag and Concurrent. Issuance of Industrial License is a Lead Indicator of future industrial activity. Similarly demand for Household Furniture in a housing complex is a Lag Indicator because it becomes active only after the construction of houses are completed i.e after a certain time lag. Concurrent Indicators move together i.e no lead or lag time is involved.
Delphi Technique/ expert opinion method	This technique is used to make more realistic judgemental forecasts by minimizing bias. In this method a panel of experts (including decision makers, staff personnel and respondents) is asked sequential questions. It is a step by step procedure and final forecast is obtained by the common opinion of all the experts.



CONCEPT – 3 ; QUALITATIVE METHODS

3.a. Regression Analysis
3.b. Time Series Analysis
3.c. Exponential Smoothing
3.b Input – Output Models

CONCEPT 3.A. REGRESSION ANALYSIS

SLR – Simple linear regression

y= a + bx

1. $\sum y = an + b.\sum x \ldots$
2. $\sum xy = a\sum x + b.\sum x^2 \ldots$

Step – 1 :calculate the values $\sum x, \sum y, \sum xy, \sum x^2$.

Step – 2 : form equations 1 & 2 . then solve to find a, b values , form equation y = a + bx .

Step – 3 : make forecasting for relative x values.

MLR – Multiple linear regression

MLR Model $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2$ calculation of the coefficients β_1 and β_2 & the intercept β_0 are done by using the formulae given below.

s.no	y	x_1	x_2	x_1^2	x_2^2	x_1y	x_2y	x_1x_2

- $\beta_0 = \bar{y} - \beta_1\bar{x_1} - \beta_2\bar{x_2}$

  $\bar{y}$ is mean value of y : $\bar{y} = \frac{\sum y}{N}$

  $\bar{x_1}$ is mean value of x_1: $\bar{x_1} = \frac{\sum x_1}{N}$

  $\bar{x_2}$ is mean value of x_2: $\bar{x_2} = \frac{\sum x_2}{N}$

- $\beta_1 = \frac{[\sum(X_2)^2.\sum X_1y - \sum X_1X_2.\sum X_2y]}{\sum(X_1)^2.\sum(X_2)^2 - (\sum X_1X_2)^2}$

- $\beta_2 = \frac{[\sum(X_1)^2.\sum X_2y - \sum X_1X_2.\sum X_1y]}{\sum(X_1)^2.\sum(X_2)^2 - (\sum X_1X_2)^2}$

- $\sum(X_1)^2 = \sum(x_1)^2 - \frac{(\sum x_1)^2}{N}$.
- $\sum(X_2)^2 = \sum(x_2)^2 - \frac{(\sum x_2)^2}{N}$
- $\sum X_1y = \sum x_1y - \frac{\sum x_1 \sum y}{N}$
- $\sum X_2y = \sum x_2y - \frac{\sum x_2 \sum y}{N}$
- $\sum X_1X_2 = \sum x_1x_2 - \frac{\sum x_1 \sum x_2}{N}$

CONCEPT - 3.b : TIME SERIES ANALYSIS

The term "*Time Series*" refers to a series of observations recorded in accordance with the time of occurrence.

Following are few examples of Time Series data –

- Profits earned by a company for each of the past ten years
- Number of students registered for CMA Examination for the past decade.
- Percentage change in quarterly Consumer Price Index (CPI) of a country for the last 45 years
- Number of employees hired by a company for each of the last five years

Components of Time Series data

The four components of Time Series are –

i. Secular Trend or Trend (T)
ii. Seasonal Variation (S)
iii. Cyclical Fluctuation (C)
iv. Irregular or Random Movement (I).

Secular Trend or simply Trend of time series is the smooth, regular and long term movement exhibiting the tendency of growth or decline over a period of time.

Seasonal Variation is a type of periodic movement where the period is at the most one year. Business activitiesare found to have brisk and slack periods at different parts of the year.

Cyclical Fluctuation is another type of periodic movement where the period is more than a year. Such movementsare fairly regular and oscillatory in nature. One complete period is called a cycle..

Irregular or Random movements are such variations which are caused by factors of erratic nature. Theseare completely unpredictable or caused by such unforeseen events as war, natural calamities, strike, lockout etc.

Measurement of Trend

There are four methods of isolating Secular Trend – (long term fluctuations) from the Time Series data.

(A) Free Hand Method
(B) Semi Average Method
(C) Moving Average Method
(D) Fitting Mathematical Curves

A. In **Free Hand Method**, the given data are plotted as points on a graph paper. The time series data (y_t) are takenalong vertical axis and time (t) along horizontal

B. **Semi Average Method** deals with dividing the dataset into two equal parts and then finding average of each. These averages are plotted as points on a graph paper against the mid-point of the time interval covered by each part. The straight line joining the two points is considered to be the trend line.

C. Moving Average Method

- Is very commonly used for the isolation of trend and in smoothing out fluctuations in time series.
- In this method a series of arithmetic means of successive observations known as Moving averages are calculated from overlapping groups of successive elements of the given data and these moving averages are used as trend values. Each moving average is based on values covering a fixed time interval called *Period of Moving Average* and is shown against centre of the first.
- When **the period of moving average is ODD** then each moving average calculated is shown against the middlemost value of the concerned group of observations.
- When **the period of moving average is EVEN** then there are two middle periods and the moving

average value is placed in between the two middle terms of the time interval it covers. So in this case the moving average value will not coincide with a period of the given time series. To synchronize the moving average values with the original data, an average of two already calculated moving averages is computed and placed them corresponding to the given time set. This method is known as Centring and the corresponding moving averages are called Centred Moving Averages

D. Method Of Fitting Mathematical Curves $y = a + bx$

- In this method an appropriate type of mathematical equation is formed , The choice of the appropriate type of equation is facilitated by a graphical representation of the data.
- If the plotted data show a straight line the equation used is $y = a + bx$
- If they show a straight line tendency on a semi logarithmic graph paper, the equation used is $\log y = a + bx$ i.e an Exponential Curve.
- If the graph shows parabola then the equation used is $y = a + bx + cx^2$.
- The constants appearing in the equations mentioned above are obtained by applying Principle of Least Square. Infact it is a special category of Least Square Regression in which the independent variable is always time.

Note :

- This method is similar to simple linear regression with time factor independent variable)
  - $y= a + bx$
  - $\sum y = an + b.\sum x \ldots 1$
  - $\sum xy = a\sum x + b.\sum x^2 \ldots 2$
  - Make $\sum x = 0$,
  - If odd no of years given take x values as ....-3, -2, -1 , 0 , 1 , 2 , 3.....
  - If even no of years are given then take x values as .... -5, -3, -1 , +1, +3, +5,.....

Measurement of Seasonal Variation.

Four methods are generally used for measuring Seasonal Variation or Short term fluctuation of time series.

- Method of Simple averages (monthly or quarterly)
- Moving Average Method (Normally Quarterly type with Monthly type at times)
- Trend Ratio Method
- Link Relative Method

Method of Simple Averages :

- Is generally applied to the time series data which do not contain trend or cyclical fluctuation to any appreciable extent.
- From the given quarterly data the averages (A_1, A_2, A_3 & A_4) for the four quarters are calculated and also Grand Average [$G = (A_1 + A_2 + A_3 + A_4)/4$] is calculated.
- **Additive Model, -** The deviations of the Quarterly Averages from the Grand Average give the measures of Seasonal Variation of the four quarters as $S_1 = A_1 - G$, $S_2 = A_2 - G$, $S_3 = A_3 - G$ and $S_4 = A_4 - G$ - (Illustration – 7)
- **Multiplicative Model -** Each Quarterly Average is expressed as percentage of Grand Average to give the Seasonal Indices for the 4 quarters as $S_1 = (A_1/G)x100$, $S_2 = (A_2/G)x100$, $S_3 = (A_3/G)x100$,$S_4 = (A_4/G)x100$ -

- When monthly figures are given, then instead of Quarterly Averages we need to calculate Monthly Averages.
- Total of Seasonal Variations = 0 for Additive Model and Average Seasonal Index = 100 for Multiplicative

Moving Average Method

- Here at first quarterly or monthly (as the case may be) Trend values are calculated using the concept of Moving Average. Thereafter the effect of Trend is eliminated from the original data.
- **For Additive Model** - this is done by subtracting Trend values from the original data to give "Deviations from Trend".
- Thereafter with these values the technique used in the Method of Simple Averages (as explained above) is applied to get the measures of Seasonal Variations. The total of these Seasonal Variations should be 0. In case that does not happen then some adjustment in the calculated values are done. (Illustration - 8)

- **For Multiplicative Model** - the effect of Trend is eliminated by finding "Ratio to Moving Average" expressed as percentage which is actually
  (Original data for a quarter or month /Moving Average value of that period) x 100.
- Thereafter the technique used in the Method of Simple Averages (as explained above) is used to find out Seasonal Indices.
- The total of these Seasonal Indices should be 400 for quarterly data and 1200 for monthly data. In case that does not happen then some adjustment in the calculated values are done.

Trend ratio method

This is applicable only for the Multiplicative type Time Series data. Here trend values are obtained by fitting a mathematical curve and the original data are expressed as percentages of corresponding trend. Thereafter the technique used for Multiplicative Model cases of Simple Average Method is utilised to get the values of Seasonal Indices of the different quarters or months.

Link Relative Method

If quarterly data are given, each value is expressed as a percentage of the value for the immediately preceding period. These are called Link Relatives (L.R).
For the first quarter it cannot be calculated because there is no data before this. Then the L.Rs are arranged by quarters and the average L.R for each quarter is found.
From these L.Rs we find Chain Relatives (C.R) by relating them to a common base. For the first quarter .C.R is taken as 100.

CONCEPT - 3.C :EXPONENTIAL SMOOTHING

In Exponential Smoothing the first forecast has to be obtained by some subjective method or by taking the average of the first few time periods.
Subsequent forecasts are then obtained by repeatedly using the $u_t = u_{t-1} + ae_t$
When a = 0, no Correction is necessary to the previous forecast. When a = 1, the new forecast will

always be the latest observation. Smoothing Coefficient (a) lies between 0 and 1 and it is selected on the basis of experiments with different values. Higher the value of a, sooner it discounts the effects of old observations.

where a is a constant known as Smoothing Constant and has a value lying between 0 and 1.

Above relation may also be expressed as

- forecast (u_t) : $u_t = u_{t-1} + ae_t$ where
- Error : $e_t = y_t - u_{t-1}$
- previous forecast ; (u_{t-1}).
- Correction ; (ae_t)

For numerical computations, above relation may be used with the following steps

1. Find the Error or Discrepancy in the latest observation from the previous forecast using

   $e_t = y_t - u_{t-1}$

2. Multiply the Error (e_t) by the Smoothing Coefficient (a) to obtain the Correction (ae_t)
3. Add the Correction (ae_t) to the previous forecast (u_{t-1}) to get forecast (u_t) for the current period t.

SMQ

Illustration-1

There are two variables that need to be studied - Exports of raw cotton and Imports of manufactured goods into India. Following dataset for 7 years is provided. What kind of regression model should be used here? What are the results of this regression? Interpret the model estimators.

	Rs ` in Crores						
Exports	42	44	58	55	89	98	60
Imports	56	49	53	58	67	76	58

Illustration-2

In order to find out the effect of Educational Qualification and Experience on the Earnings of the workers of a CNC Machine Shop, a study has been conducted on 10 workers and the dataset below is provided to you.

Worker	1	2	3	4	5	6	7	8	9	10
Education	11th Std	11th Std	12th Std	12th Std	1stYr. Bach.	2ndYr Bach.	2nd Yr Bach.	1st Yr. Masters	1st Yr. Masters	1st Yr. Masters
Experience (Years)	10	6	10	5	5	6	5	8	7	2
Salary (` per month)	30000	27000	20000	25000	29000	35000	38000	40000	45000	28000

Conduct a Multiple Regression Analysis considering the following numerical equivalents for

Educational Qualification – 11th Std. = 11, 12th Std. = 12, 1st Yr Bachelors = 13, 2nd Yr. Bachelors = 14, 1st Yr. Masters = 16 Also interpret the meaning of the coefficients of the variables and the constant term.

Illustration-3

The CNC Machine Shop mentioned in the problem above is going to bid for few very critical machining items required for a Nuclear Power project. For that they have to recruit a dedicated programmer for the machines. Besides they need to procure two new CNC Vertical Machining centres which have to be operated by qualified operators. The profile they are looking for the programmer is Graduate Engineer with 5 years of experience and that for the operators of the new machining centres is 2nd year Bachelors' degree holder with 8 years of working experience. If 1000 nos. of the critical items are to be supplied within a period of 3 years then how much extra the company should bid for each item to accommodate the recruitments of a programmer and two operators. Use the dataset of the previous illustration. [Assume educational qualification of a Graduate Engineer = 20 and that of an operator with 2nd year Bachelors' degree = 14]

Illustration-4 (Moving Average trend with EVEN Period)

Find trend values of the following year wise data of Goods carried by a fleet of trucks of a Transport Company having pan India network using the Moving Average Method. [Assume a 4 yearly cycle]

Year	1975	1976	1977	1978
Goods carried (Tons)	2204	2500	2360	2680
Year	1979	1980	1981	1982
Goods carried (Tons)	2424	2634	2904	3098
Year	1983	1984	1985	1986
Goods carried (Tons)	3172	2952	3248	3172

Illustration-5 (Mathematical Curve fitting with data for ODD number of years given)

The following table relates to the tourist arrivals in India during 1990 to 1996.

Year	1990	1991	1992	1993	1994	1995	1996
Tourist arrivals (lakhs)	18	20	23	25	24	28	30

Fit a Straight Line trend by the Method of Least Squares and estimate the number of tourists that would arrive in the year 2000.

Illustration-6 (Mathematical Curve fitting with data for EVEN number of years given)

From the following past data of Sales (in lakhs Rupees) of a company estimate the same for the year 2005.

Year	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999
Sales	15.3	14.6	16.8	17.3	17.2	20.9	22.3	20	23.1	24.5

Assume the trend line to be linear. What is the monthly rate of increase of Sales?

Illustration-7

Calculate the Seasonal Indices for the following quarterly data in certain units. Appropriate method for finding the Indices has to be decided by you with due explanation.

Year	**1st Quarter**	**2nd Quarter**	**3rd Quarter**	**4th Quarter**

1992	39	21	52	81
1993	45	23	63	76
1994	44	26	69	75
1995	53	23	64	84

Illustration-8

Calculate Seasonal Fluctuation from the following Time Series data obtained from a Mini Steel Plant

	Quarterly Output of Steel in '000 Tons			
Year	**1st Quarter**	**2nd Quarter**	**3rd Quarter**	**4th Quarter**
2005	65	58	56	61
2006	68	63	63	67
2007	70	59	56	52
2008	60	55	51	58

Use 4 Quarter Moving Average Method. Consider Additive Model for the Time Series data.

Deseasonalise the given dataset.

Illustration-9

On the basis of quarterly Sales (in ` Lakhs) of a certain commodity for the period 2001 to 2005 the following calculations were made.

Trend:- Straight Line trend equation is y = 25 + 0.6t, Origin – 1st Quarter of 2001, t unit – 1 Quarter, y – Sales Seasonal Variations:-

Quarter	I	II	III	IV
Seasonal Index	90	95	110	105

Estimate the Quarterly Sales figures for the year 2010

Illustration-10

M/S B.P.Leathers, a shoe manufacturer has modern outlook and they depend heavily on Business Forecasting methodology to plan their business activities like manufacturing, marketing, finance etc. At the beginning of the year 2022 they have forecasted data of demand of their shoes for the beginning of the month of March as 1000 pairs. But the actual demand turned out to be 900 pairs. Using a Smoothing Coefficient of 0.1 forecast the demand at the beginning of the 2nd week of March 2022.

Also forecast the demands using Exponential Smoothing technique at the beginning of each week till mid April 2022 when the actual demands are as follows –

At the beginning of the 2nd week of March – 1010 pairs, At the beginning of the 3rd week of March – 1032 pairs, At the beginning of the 4th week of March – 976 pairs, At the beginning of the 1st week of April – 934 pairs, At the beginning of 2nd week of April – 1008 pairs & At the end of the 2nd week of April – 1020 pairs.

Multiple Choice Questions

1. In Exponential Smoothing Method which one of the following is true?
   (a) $0 \leq a \leq 1$ and high value of a is used for stable demand.
   (b) $0 \leq a \leq 1$ and high value of a is used for unstable demand.
   (c) $a \geq 1$ and high value of a is used for stable demand.
   (d) $a \leq 0$ and high value of a is used for unstable demand.
2. Which of the following is not a Casual Forecasting Method?
   (a) Trend adjusted Exponential Smoothing
   (b) Econometric models
   (c) Linear Regression
   (d) Multiple Regression
3. Which of the following is a Forecasting technique?
   (a) PERT / CPM
   (b) Exponential Smoothing
   (c) Gantt Chart
   (d) Control Chart
4. The number of averaging period in the Simple Moving Average Method of forecasting is increased forgreater smoothing but at the cost of –
   (a) Accuracy
   (b) Stability
   (c) Visibility
   (d) Responsiveness to changes
5. In a Time Series forecasting model, the demands for five time periods are 10, 13, 15, 18 and 22. A linear regression fit resulted in the equation $y_t = 6.9 + 2.9t$, where y_t is the forecast for the period t. The sum of theabsolute deviations for the five data with respect to their corresponding forecasts (taking t = 1 for the first one) is
   (a) 2.3 (b) 0.2 (c) 1.2 (d) 2.2
6. Which of the following is not a part of Quantitative type of Forecasting Model
   (a) Moving Average
   (b) Simple Average
   (c) Delphi Method
   (d) Exponential Smoothing
7. Which of the following Forecasting technique uses three types of participants: Decision Makers, Staff personnel and Respondents?
   (a) Expert's Opinion
   (b) Sales Force Survey
   (c) Consumer Survey
   (d) Delphi Method
8. Sales data for the numbers sold for a particular product during January to May 2007 shows the values 10, 11,16, 19 and 25. Regarding forecast for the month of June which one of the following statement is true?
   (a) Moving Average will forecast a higher value compared to regression.

(b) Exponential Smoothing will forecast a higher value compared to regression

(c) Regression will forecast a higher value compared to moving average.

(d) None of the above.

9. The Time Series forecasting method that gives equal weightage to each of the N most recent observationsis –

(a) Moving Average Method

(b) Exponential Smoothing with linear Trend

(c) Triple Exponential Smoothing

(d) None of the above

10. Which of the following is not a forecasting technique?

(a) Trend line estimate (c) Hungarian Method

(b) Delphi Method (d) Judgemental technique

11. In Simple Exponential Smoothing forecast, to give higher weightage to recent demand information, thesmoothing constant must be close to –

(a) -1 (b) 0 (c) 0.5 (d) 1

12. Which of the following is not true for forecasting?

(a) Forecasts are rarely perfect.

(b) The underlying casual system will remain same in the future.

(c) Forecast for group of items is accurate than individual item

(d) Short range forecasts are less accurate than long range forecasts.

13. In which of the following forecasting technique, data obtained from past experience is analysed?

(a) Judgemental forecast (c) Associative model

(b) Time Series forecast (d) All of the above

14. Delphi Method is used for –

(a) Judgemental forecast (c) Associative model

(b) Time Series forecast (d) All of the above

15. Short term regular variations related to the calendar or time of the day is known as –

(a) Trend (c) Cycles

(b) Seasonality (d) Random variations

16. A linear Trend equation has the form –

(a) F = a – bt (c) F = 2a – bt

(b) F = a + bt (d) F = 2a + bt

17. The actual demand for a period is 100 units. But forecast demand was 90 units. The forecast error is –

(a) – 10 (b) 10 (c) 5 (d) None of the above

18. Which of the following is not a forecasting technique?

a) Judgemental c) Time Horizon

b) Time Series d)Associative

19. Which of the following is not a Qualitative Forecasting technique?

(a) Surveys of consumer expenditure plans

(b)Perspective of foreign advisory councils

(c) Consumer intention polling

(d) Time Series analysis

20. Which of the following is not one of the four types of variation that is estimated in the Time Series analysis?

(a) Predictable (c) Cyclical

(b) Trend (d) Irregular

21. In Time Series Analysis which source of variation can be estimated by the ratio to trend method?

(a) Cyclical (b) Trend (c) Seasonal (d) Irregular

22. A qualitative forecast

(a)Predicts the quality of a new product

(b)Predicts the direction but not the magnitude of change in a variable.

(c)Is a forecast that is classified on a numerical scale from 1 (poor quality) to 10 (perfect quality).

(d)Is a forecast that is based on econometric methods.

23. The first step in Time Series analysis is to –

(a)Perform preliminary Regression calculations.

(b)Calculate a moving average.

(c)Plot the data on a graph

(d)Identify relevant correlated variables.

24. If the estimate of the Trend Component is 158.2, the estimate of Seasonal Component is 94%, the estimate of the Cyclical Component is 105% and the estimate of the Irregular Component is 98%, then the multiplicative model will produce a forecast of –

(a) 1.53 (b) 1.53% (c) 153.02 (d) 153,020,532

25. From the first two supplied values of a Time Series and its corresponding Exponential Smoothing forecast as given below, the forecast for the time period 3, assuming Smoothing Constant = 0.3, will be

Time Period (t)	Value (y_t)	Exponential Smoothing Forecast (u_t)
1	18	18

2	22	18

(a) 18 (b) 19.2 (c) 20 (d) 40

26. For the Regression Equations y = 0.516x + 33.73 and x = 0.512y + 32.52, the Arithmetic Mean values of x and y are nearly

(a) 67.6 and 68.6 (c) 67.6 and 58.6
(b) 68.6 and 68.6 (d) 68.6 and 58.6

27. For a bivariate dataset (x,y), if the Means, Standard Deviations and Correlation Coefficient are respectively x = 1, y = 2, σ_x= 3, σ_y = 9 and r = 0.8. The regression line y on x is-

(a) y = 1 + 2.4(x -1) (c) y = 2 + 2.4(x – 1)
(b) y = 2 + 0.27(x – 1) (d) y = 1 + 0.27(x – 2)

28. The data about Sales and Advertisement Expenditure of a firm is given below

	Sales (` Crores)	**Advertisement Expenditure (` Crores)**
Mean	40	6
S.D	10	1.5

The Correlation Coefficient between Sales and Advertisement Expenditure is 0.9. The likely Sales for a proposed Advertisement Expenditure of ` 10 Crores is-

(a)` 64 Crores (b) ` 67 Crores (c) ` 70 Crores (d)` 58 Crores

29. Given the Regression Lines x + 2y – 5 = 0 and 2x + 3y – 8 = 0 and Var (X) = 12. The value of Var (Y) is –

(a) $\frac{3}{4}$ (b) 4/3 (c) 16 (d) 4

30. The equations of the two lines of Regression are 4x + 3y + 7 = 0 and 3x + 4y + 8 = 0. The Coefficient of proposed Advertisement Expenditure of ` 10 Crores is-

(a) 1.25 (b) 0.25 (c) – 0.75 (d) 0.92

Answers:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
b	a	b	d	d	c	d	c	a	c	d	d	b	a	b
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
b	b	c	d	a	c	b	c	c	b	a	c	a	d	c